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A PARTIAL EFFECT SIZE FOR THE SYNTHESIS OF MULTIPLE REGRESSION MODELS

By

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For my parents and sisters, Hugo Luis Aloe, Ana Maria Rozas de Aloe, Evangelina Lujan Aloe and Lorena Martha Aloe

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ABSTRACT

A new approach to representing data from multiple regression designs is presented in this dissertation. The index, denoted as r_{sp}, is the semi-partial correlation of the predictor with the outcome of interest. This effect size can be computed when multiple predictor variables are included in the regression model, and represents a partial effect size in the correlation family. The derivations presented in this dissertation provide the partial effect size and its variance. Standard errors and confidence intervals can be computed for individual r_{sp} values. Also, meta-analysis of the semi-partial correlations can proceed in a similar fashion to typical meta-analyses weighted analyses can be used to explore heterogeneity and to estimate central tendency and variation in the effects. A simulation study is presented to study the behavior of this index and its variance.

CHAPTER 1

INTRODUCTION

The majority of study designs and statistical analyses used these days control for the effects of multiple variables (e.g., individual characteristics) in the model. Kerlinger (1979) argued that to understand contemporary behavioral research one needs a real good understanding of multivariate techniques. However, most meta-analysis techniques focus on the synthesis of bivariate relationships. This is in part because methods to synthesize multivariate analyses, including multiple regression models, are not yet well understood. In addition, several complications arise due to the characteristics of multiple regression analysis. For instance, different models include different numbers of predictors, and the predictors of interest are measured with different instruments and scales across the primary studies. Typically, the meta-analyst will want to synthesize data on relationships involving a predictor of interest and an outcome, taking into consideration the influence of other predictors in the model.

Meta-analysis techniques for the synthesis of univariate outcomes are well understood. However, in the last few decades the number of primary studies using multivariate techniques has increased notably, making simpler univariate approaches to meta-analysis more difficult to apply and justify. Consequently, researchers conducting quantitative reviews face the alternatives of omitting a large number of multivariate primary studies, or attempting to synthesize those multivariate studies and, in some cases, combining their results with ones obtained from univariate designs. However, when dealing with effects from multivariate designs such as multiple regression models, there is no universal approach on which meta-analysts agree. Thus new indices of effect magnitude, and methods of synthesizing them, are called for.

Lack of Agreement about Methods for the Synthesis of Regression Results

Often meta-analysts have had to omit primary studies that used multiple regression models. For example, in a meta-analysis concerning the effects of psycho-social determinants in pro-environmental behaviors Bamberg and Möser (2006) stated, "During this [data evaluation] step we lost a substantive number (*n* = 31) of interesting studies (e.g., Chu & Chiu, 2003; Oom do Valle, Rebello, Reis, & Menezes, 2005; Thøgersen, 2006) because they only reported multivariate results obtained from regression or SEM analyses without documenting the respective bivariate correlations. Forty-six studies reporting results for 57 independent samples fulfill all the selection criteria." (Bamberg & Möser, 2006, p. 17). Another common practice is to transform other statistics reported in primary studies, such as transforming *t* tests into zero-order correlations or partial correlations. However, often authors ignore the difficulties that arise from such practices. For instance, effect sizes computed from some of these transformations are in a different metric than the Pearson's product-moment correlation coefficient (*r*), as I will show below.

The problem that occurs in the synthesis of multiple regression models is an important one. Different reviewers deal with the practical issue of multiple regression models in primary studies in different ways. When primary studies do not report correlations between the predictor and outcome of interest, reviewers often attempt to obtain correlations from what is reported in studies. A typical approach is to try to obtain *r* from other statistics reported. For instance, Card and Little (2006) stated that "Effect sizes were represented as Pearson correlations, *r*. For studies reporting results in other metrics, these data were transformed to *r* using standard procedures (e.g., Rosenthal, 1991)" (p. 468). Unfortunately, there are no such standard procedures for transforming other forms of statistics into *r* with the exception of the transformation from the *d*-family of effect sizes. Another common statement found in the literature is that partial correlations were obtained from multiple regression models. For instance, Welten et al. (1995) stated … "the most commonly reported measure of effect was the Pearson's product-moment correlation coefficient (*R*). A second effect measure used in this meta-analysis was the partial correlation coefficient (*PR*), derived from multiple regression analysis….When this was not given, *PR* was calculated from corresponding *t* value, *F* value or estimate otherwise from the *P* value" (p. 2804). It is clear that these researchers are combining different types of effects in their meta-analyses. Some other examples follow.

"The first-order effect sizes that were extracted included standardized regression coefficients, complete and semipartial correlations, and the square root of the differences in R^2." (Haring-Hidore et al., 1985, p.950).

"The first step in the calculations involved computing the effect size estimate (the product-moment correlation coefficient *r*, see Rosenthal, 1991)…. We computed the effect sizes either from means and standard deviations, if reported, or otherwise from the statistics (e.g., *t*, *F*)." (Phaf & Kan, 2006, p. 189).

"The Pearson's product-moment correlation coefficient, *r*, was chosen as an appropriate measure of effect size… coefficients were obtained for each study, when possible, in the following ways (in order of preference): (i) direct reporting of *r*, R^2, or partial correlation; (ii) mean and variance data (s.e. or s.d.) reported in the text or figures converted to *r* using methods in Rosenthal (1991); (iii) other test statistics (e.g., *F*, *U*, *t*, χ^2) converted to *r* using methods in Rosenthal (1991); (iv) exact *p*-values converted to *r* using META-ANALYSIS 5.3" (Stankowich & Blumstein, 2005, p. 2).

Purpose of This Dissertation

The overall goal of this dissertation is to develop and evaluate statistical techniques for the meta-analysis of primary studies that use multiple regression models. Multiple regression analyses have been used in the literature as a statistical tool to understand the relationships among a dependent variable and multiple predictors. For instance, multiple regression analysis is widely used in the literature concerning teachers' qualifications (e.g., type of certification, teachers' verbal ability) and students' academic achievement (e.g., overall GPA, standardized tests). A new approach to representing data from multiple regression designs is examined in this dissertation. The proposed index r_{sp} is the semi-partial correlation of the predictor with the outcome of interest. This effect size can be computed when multiple predictor variables are included in each model, and represents a partial effect size in the correlation family. Throughout this dissertation I use examples drawn from an on-going synthesis of relationships between measures of teacher verbal ability and school outcomes (Aloe & Becker, 2008).

Specifically, Chapter 2 contains a literature review. Chapter 3 includes the derivations. Chapter 4 presents an example of the synthesis of r_{sp} values. Chapter 5 presents the simulation study. Chapter 6 of the dissertation presents the results from the simulation designs. Finally, Chapter 7 presents the discussion.

CHAPTER 2

LITERATURE REVIEW

Most of the existing methods that deal with the synthesis of multiple regression models are based on synthesis of the slopes. In this section I first introduce the product moment correlation (*r*). Second, I discuss the problems in combining regression models (Becker & Wu, 2007). Next, I discuss indices proposed to represent slopes such as the raw slope, standardized slope, ubiquitous effect size (Timm, 2004), and a partial effect size for the *d* family obtained from regression models with a dummy variable (Keef & Roberts, 2004). Finally, I discuss the transformation from *t* to *r* presented by Rosenthal (1991) and McCarthy and Rosenthal (2001).

Correlations

Effect sizes should be scale free indexes which assess the magnitude of the relationship between an independent variable and a dependent variable. My focus is on the use of effect sizes from the *r* family (e.g., the product moment correlation and point biserial correlation). The effect sizes represented by the *r* family can be interpreted as representing the strength of relationship between two (usually continuous) variables.

The product moment correlation *r* is one of the effects most typically used in meta-analysis. Let *n* be the number of observations of two variables *X* and *Y* which are continuous and bivariate normally distributed. The correlation formula can be written in many different ways; introductory statistics books often offer some formula algebraically equivalent to

$$r = \frac{S_{XY}}{S_X S_Y}, \qquad (2.1)$$

where S_{XY} is the covariance between *X* and *Y*, and S_X and S_Y are the standard deviations for *X* and *Y* respectively.

The large sample variance of *r* (see, e.g., Olkin & Siotani, 1976) is var(*r*) =

$(1-\rho_{XY}^2)^2/(n-1)$, and the expected value of r is ρ_{XY} in large samples (though r is a biased estimator in small samples). Furthermore it is common to treat r as normally distributed in large samples, and I will use this result below in deriving an asymptotic normal distribution for the proposed estimator. Typically, the variance of r is estimated by substituting r for ρ in var(r). This is important for meta-analysis because it enables the reviewer to compute an estimate of the uncertainty of each correlation in a review, using only the sample size and the sample value of r.

Problems in Combining Regression Models

Primary studies tend to report simpler components of multivariate results, such as canonical correlations, or slopes, t statistics, and standard errors, and to omit more complex details such as full correlation matrices, or variance-covariance matrices among slopes. When full covariance or correlation matrices are reported, it may be possible to apply Becker's (1992) techniques for synthesis. However, reporting these matrices is more the exception than the rule. Therefore we need to find effect indices that can be computed based on other summary statistics that are likely to appear in study reports taking into account the increased methodological difficulties described below. Becker and Wu (2007) discussed some of the difficulties of combining regression analyses. The authors discussed the influence of other Xs in the model, multicollinearity, and scaling of X and Y.

Other Xs in the model. One of the major difficulties in the representation and synthesis of effects obtained from regression models arises because of additional predictors (Xs) across studies (Becker & Wu, 2007). When a regression model is well specified, it should provide an unbiased estimate of the slope of interest, which in turn represents the relationship between its associated predictor and the outcome Y. However, in a multiple regression model, each slope does not represent a simple bivariate X-Y relation. In addition, each slope is a partial regression slope, thus the interpretation of the slope of, say, X_p in a p predictor model is that it shows the effect of X_p on Y, controlling for (or "holding constant") X_1 through X_{p-1}.

Having different Xs in the models that arise across studies can lead to apparent variation in the strength of relationship between the predictor of interest and the outcome (that is, variation in the observed slopes), particularly if the *X*s are interrelated. In addition, different primary studies typically estimate different models. This is because in general researchers develop new models by adding or subtracting variables from existing ones. In practice the presence of different predictors across models is almost always expected, given that researchers are always trying to improve upon previous research. As a result, when conducting a meta-analysis, it is common to find one's predictor of interest in models with a variety of other *X*s.

How common is diversity in the predictors that appear in regression models examining a particular relationship? In Aloe and Becker's (2008) synthesis of teacher science knowledge, five studies reported regression analyses (but no correlation coefficients). As few as three and as many as nine measures of teacher knowledge were included in the models that were analyzed, and some of those appear likely to be correlated with each other (though evidence of multicollinearity of the predictors was not discussed in any of the reports). The full models included between 7 and 27 predictors. It is clear that to restrict the synthesis to only those studies reporting correlations would eliminate more-sophisticated analyses, and more complex and presumably more realistic models of science achievement.

In another recent meta-analysis Aloe and Becker (2008) synthesized studies of the relationship between teachers' verbal ability and school outcomes. Fifteen effects were obtained from studies that reported multiple regression models. The 15 models from which those effects were drawn were all different. Although verbal ability and a few other predictors appeared in all of the models, some of the models contained seven *X*s and others more than twenty. Because none of the reports included a full correlation matrix among the variables examined, it is not clear how intercorrelated the other predictors were with verbal ability.

Multicollinearity. One particular concern in any multiple regression is the extent to which the predictors are interrelated, that is, the extent of multicollinearity (Becker & Wu, 2007). Ideally, all independent variables in such models will be highly correlated with the outcome but uncorrelated with each other. However, in reality this almost never happens because viable predictor variables in education and psychology are usually more or less correlated (Meehl, 1990). Even so, it is rare for primary-study authors to report on multicollinearity, or to give the correlations among predictors in their analyses. As I will show below, the extent to which a predictor of interest correlates with other predictors will affect the proposed partial effect-size index.

Scaling of X and Y. In addition to the fact that having different sets of predictors leads to different interpretations of slopes and to variation in the observed strength of relationship as indexed by the regression slope, one other issue must be considered in selecting an effect size for results from multiple regression analyses (Becker & Wu, 2007). That is, both predictors and outcomes may be measured using different scales across studies. The scales are presumably meant to represent a common construct or latent variable, but the fact that the measures themselves may differ means that raw regression coefficients will often not be comparable. In some cases scales can be considered comparable or translated to be comparable – such as when the variables represent money, time, or physical measures such as height or weight. But for many variables in education and the social sciences there are no "natural" or true scales, thus making the raw regression slope a poor choice for an effect size. Optimally any effect size used to represent regression results should be scale free.

Proposed Indices to Represent Slopes

In this section I discuss the proposed indices to synthesize slopes such as the raw slope, standardized slope, ubiquitous effect size, and a partial effect for the *d* family obtained from regression models with a dummy variable.

Raw Slope

The least squares regression model can be written as $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{e}$, where the outcome $\mathbf{Y} = (Y_1, Y_2, \ldots, Y_n)'$ for a sample size n, the $n \times p$ matrix of predictors $\mathbf{X} = (\mathbf{X}_1, \ldots, \mathbf{X}_p)'$, the slope parameter vector $p \times 1$ is $\boldsymbol{\beta} = (\beta_0, \beta_1, \ldots, \beta_p)'$ and the vector of errors $\mathbf{e} = (e_1, e_2, \ldots, e_n)'$, which is normally distributed with a mean of zero and variance covariance matrix $\boldsymbol{\Sigma}\sigma^2$. When the typical assumptions hold the slope is estimated as

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}'\boldsymbol{\Sigma}^{-1}\mathbf{X})^{-1}\mathbf{X}'\boldsymbol{\Sigma}^{-1}\mathbf{Y} . \tag{2.2}$$

When the matrix $\boldsymbol{\Sigma}$ is the identity matrix the ordinary least squares (OLS) estimate of $\boldsymbol{\beta}$ is obtained. In addition, the variance covariance matrix of the slope is

$$\operatorname{var}(\hat{\boldsymbol{\beta}}) = (\mathbf{X}'\boldsymbol{\Sigma}^{-1}\mathbf{X})^{-1}\sigma^2 . \tag{2.3}$$

The slope can also be written as a function of the correlations among the predictors in **X**, their correlations with **Y**, and their variances. Specifically, for a model with two predictors the slope of X_1 is

$$b_1 = \left(\frac{r_{Y1} - r_{Y2}r_{12}}{(1 - r_{12}^2)}\right)\frac{S_Y}{S_1} \tag{2.4}$$

where r_{Y1} is the correlation of the outcome with the predictor X_1, r_{Y2} is the correlation of the outcome with the predictor X_2, r_{12}^2 is the squared correlation of the predictor X_1 with the predictor X_2, S_1 is the standard deviation of X_1, and S_Y is the standard deviation of Y.

Becker and Wu (2007) noted that several conditions must hold to synthesize raw slopes across studies. Particularly, measures of the predictor of interest X and the outcome Y must be on the same metric (scale) across studies, and the same additional models should be included in all regression models that are to be summarized. In practice, it is virtually impossible to meet these conditions since different measures and models are typically used in primary studies.

Standardized Slope

The standardized slope (or beta weight) has been used as an index in meta-analysis studies (Cooper et al., 2006; Farley et al., 1981). The standardized slope can be interpreted as the number of standard deviation units of change predicted to occur in Y, given a one standard deviation unit change in the predictor. This index can be estimated as

$$b_1^* = \frac{r_{Y1} - r_{Y2}r_{12}}{1 - r_{12}^2}, \tag{2.5}$$

where r_{Y1} is the correlation of the outcome with the predictor X_1, r_{Y2} is the correlation of the outcome with the predictor X_2, and r_{12}^2 is the squared correlation of the predictor X_1

with the predictor X_2. When the predictors are uncorrelated ($r_{12} = 0$) the formula reduces to the zero-order correlation coefficient. One of the major shortcomings of synthesizing standardized slopes is that generally primary researchers report beta weights without standard errors.

Timm's Index (Ubiquitous Effect Size)

Timm (2004) defined the linear model as $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{e}$ with a matrix of predictors $\mathbf{X}$ that is $n \times p$, a $p \times 1$ parameter vector $\boldsymbol{\beta}$, and with the typical error assumptions $e_j \sim N(0, \sigma^2)$ for j = 1 to n. Specifically, Timm's index for this model can be defined as

$$\Delta_T = \frac{[(\boldsymbol{\Psi} - \varphi_0)'[\mathbf{C}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{C}']^{-1}(\boldsymbol{\Psi} - \varphi_0)]^{\frac{1}{2}}}{\sigma\sqrt{(n-p)+q+1}}, \qquad (2.6)$$

where $\boldsymbol{\Psi} = \mathbf{C}\boldsymbol{\beta} = \varphi_0$, $\mathbf{C}$ is a $q \times p$ matrix that defines a set of linear combinations of the slopes β_0 through $\beta_{p\text{-}1}$. I am not aware of any meta-analysis that has yet implemented Timm's index. According to the ISI Web of Science, as of late 2008 only two articles have cited Timm (2004). Neither of the two articles are synthesizing ubiquitous effect-size indexes.

Synthesis of *t* Tests

Stanley and Jarrell (1989, 2005) recommended the synthesis of *t* tests of the slope. According to the authors, the issue of the different metric in primary studies is removed by dividing the slope by its standard error. In addition, the authors argued that the synthesis of *t* values is a way to deal with heteroskedascity of slopes among studies. Heteroskedascity arises given the different sample sizes among studies. Unfortunately, Stanley and Jarrell did not specify how to summarize the *t* values or how one could obtain a variance for this index.

A Partial Effect Size for the *d* Family

In spite of the fact that primary studies have become increasingly complex and multivariate, relatively little has been done to examine partial effect sizes in the context of meta-analysis. A key exception to this is the work of Keef and Roberts (2004). Keef and Roberts considered the case of treatment effects on continuous variables, and proposed the use of a partial standardized mean difference for this situation. Keef and Roberts called upon arguments made by Cooper and Hedges (1994a) and Becker and Schram (1994), stating that the synthesis of more complex models (and more complex study results) is likely to become increasingly important.

Keef and Roberts examined the case of the partial *d*-type (mean-difference) effect size, where a two-group comparison is examined by way of an analysis-of-covariance model. Specifically, their model for case *j* was

$$Y_j = \alpha + \gamma D_j + \beta_2 X_{2j} + \ldots \beta_p X_{pj} + e_j,$$

where *Y* is an outcome score, *D* is a dummy variable representing a treatment or group effect, and X_2 through X_p are covariates. The errors e_j are assumed to have common variance σ^2. Keef and Roberts proposed using $g_{\text{adj}} = \hat{\gamma} / \hat{\sigma}$ as a partial index of treatment effects, since $\hat{\gamma}$ represents an adjusted mean difference (accounting for all covariates in the model) and $\hat{\sigma}^2$ is the residual variance – essentially the variance of the *Y* scores, partialling out the effects of all predictors.

It is not possible to predict whether a *d*-type partial effect size will be smaller or larger than the zero-order effect size without specific information about the data at hand. Because two adjustments are at play – the adjustment to the mean difference and also a reduction in the standard deviation – g_{adj} can be either smaller or larger than the unadjusted effect size. It can be larger than the typical standardized mean difference if the adjusted mean difference does not differ much from the unadjusted mean difference, but the standard deviation $\hat{\sigma}$ is much smaller than the unadjusted standard deviation S_Y. However, if the adjustment leads to a greatly reduced mean difference, but the residual standard deviation is not much smaller than the unadjusted standard deviation, g_{adj} could be smaller than a zero-order effect size.

Transforming Slope *t* Tests to *r* Using Rosenthal's Approach

In an effort to include the largest number of studies relevant to the topic of interest in meta-analysis, researchers have used other approaches to estimate *r*-family effect sizes from regression studies. Some interest has been shown in using the *t* test of the slope to represent the relationship between two variables in meta-analysis. McCartney and Rosenthal (2000) stated that it is possible to obtain a partial correlation (say, r_p) from multiple regression analysis using the t test of the regression coefficients by computing

$$r_p = \frac{t_b}{\sqrt{t_b^2 + df}}, \tag{2.7}$$

where $df = n - p - 1$ and t_b is a t test of a regression slope. However, Lipsey and Wilson (2001) stated that formula 2.7 is only appropriate when the t statistic is from a test of $\rho = 0$.

Unfortunately, the literature contains no information regarding the behavior of this transformation for other kinds of t values. For instance, it is not completely clear under what conditions (if any) the r_p obtained from formula 2.7 is a partial correlation, whether it can ever equal a zero-order r (i.e., one computed directly from raw data), or whether it ever would equal the value of r_{sp}. As was noted in chapter 1, several researchers have stated that partial correlations were obtained from formula 2.7. In the next section the partial and semi-partial correlation are defined.

Partial correlation. When a third variable is held constant or partialled out of two variables that are correlated we obtain a partial correlation. The partial correlation formula for the case of two predictors can be written as

$$r_{Y1\bullet 2} = \frac{r_{Y1} - r_{Y2}r_{12}}{\sqrt{(1 - r_{Y2}^2)(1 - r_{12}^2)}}, \tag{2.8}$$

where r_{Y1} is the correlation between Y and X_1, r_{Y2} is the correlation between Y and X_2, and r_{12} is the correlation between X_1 and X_2.

Semi-partial correlation. Estimating effect sizes from multiple regression models is one of the situations in which it may be of interest to partial out a independent variable from other predictors in the equation. For instance, if the researcher is interested in the increment in the proportion of variance in students' academic achievement accounted for by teachers' verbal ability, this is represented by the semi-partial correlation. A semi-partial correlation is a correlation between a dependent variable and an independent variable from which another independent variable has been partialled out. Specifically,

$$r_{\text{sp}} = r_{Y(1\bullet 2)} = \frac{r_{Y1} - r_{Y2}r_{12}}{\sqrt{(1 - r_{12}^2)}}, \tag{2.9}$$

where r_{Y1}, r_{Y2}, and r_{12} were defined above. When $r_{12} = 0$ formula 2.9 is equal to the bivariate correlation. On the other hand when $r_{12} = 0$ the partial r in formula 2.8 reduces to

$$r_{Y1\bullet 2} = \frac{r_{Y1}}{\sqrt{(1 - r_{Y2}^2)}}. \tag{2.10}$$

Thus, even when the two predictors in the model are completely uncorrelated, the simplified partial correlation show in formula 2.10 doesn't equal the bivariate correlation ($r_{Y1\bullet 2} \neq r_{Y1}$).

CHAPTER 3

DERIVATIONS

In this section I first introduce a partial effect for the r family (r_{sp}). Second, the variance of the r_{sp} index is derived. Third, linear models with two predictors are discussed. Next, the relationship among partial and semi-partial correlations and r is discussed. Finally, linear models with multiple predictors are discussed.

A Partial Effect Size for the *r* Family

The proposed index (r_{sp}) is to be used to represent results of multiple regression analyses with continuous predictors. This index represents the partial association between the dependent variable (Y) and a predictor of interest (say X_p) controlling for the effects of other predictors in the model. In this case the model

$$Y_j = \beta_0 + \beta_1 X_{1j} + \beta_2 X_{2j} + \ldots \beta_p X_{pj} + e_j$$

is similar to that of Keef and Roberts (2004), but our focal predictor is not a dummy variable – instead it is continuous. I will refer to the variable X_p as the focal predictor, and the index of effect will be the semi-partial correlation r_{sp} between X_p and Y.

The semi-partial correlation index is the correlation of Y with the part of X_p that is uncorrelated with all other Xs. With two predictors, when X_1 and X_2 are unrelated, $r_{\text{sp}}^2 = r^2$. However, if X_1 and X_2 are correlated it is most typical that r_{sp} will be smaller than the zero-order r value. Below I show this result algebraically as well.

Pedhazur (1997) and others have pointed out that the squared semi-partial correlation can be expressed as the difference between two R^2 (“variance explained”) values. Thus, if we define the focal X to be the pth predictor variable, we can write the formula for r_{sp} as

$$r_{\text{sp}} = \text{sgn}(b_p)\sqrt{r_Y^2 - r_{Y(p)}^2}\ , \qquad (3.1)$$

where r_Y^2 is the total variance explained by the model with p predictors, $r_{Y(p)}^2$ is the variance accounted for by the model with p - 1 predictors (i.e., without the focal predictor X_p), b_p is the slope of the predictor X_p, and sgn(b_p) indicates that the sign of r_{sp} (positive or negative) will be the same as that of the slope. The total variance in Y explained by X_1 through X_p minus the variance explained by the set of $p - 1$ variables (omitting X_p) equals r_{sp}^2.

Unfortunately, it is rather unusual to find one of the components of formula 3.1 – the R^2 for the reduced model excluding the focal predictor – in reports of regression model results. However, the semi-partial correlation can be obtained from other results that are typically reported for multiple regression models. It can be computed as (Pedhazur, 1997)

$$r_{sp} = \frac{t_p \sqrt{(1 - r_Y^2)}}{\sqrt{(n - p - 1)}}, \qquad (3.2)$$

where t_p is again the t test of the regression coefficient β_p (i.e., the test of H_0: $\beta_p = 0$) in the multiple regression model, r_Y^2 is the total variance accounted for by the full model with p predictors, and n is the number of participants.

The Variance of r_{sp}

Nearly all effect-size indices used in meta-analysis have variances that can be computed from sufficient statistics, and that represent the uncertainty in the effect-size values. I next present the variance of the r_{sp} index.

Hedges and Olkin (1981) derived the asymptotic distribution of commonality and uniqueness values, through which it is possible to obtain the asymptotic variance of the difference in R^2 values $r_Y^2 - r_{Y(p)}^2$, which is the square of the r_{sp} index. The major difficulty in applying Hedges and Olkin's distribution is that a complex matrix is needed to compute the covariances among the elements. Fortunately Alf and Graf (1999), motivated by Olkin and Finn (1995), developed simplified equations for the asymptotic variance of the difference between two squared multiple correlation coefficients, that is, for $r_Y^2 - r_{Y(p)}^2$. Consequently, the derivation of the variance of r_{sp} (which is the square root of $r_Y^2 - r_{Y(p)}^2$) can be achieved via a straightforward application of the delta method (e.g., Rao, 1973).

Let the function of interest r_{sp} be expressed as a function of the values $(r_Y, r_{Y(p)})$, as $r_{\text{sp}} = f(r_Y, r_{Y(p)}) = \text{sgn}(b_p)\sqrt{r_Y^2 - r_{Y(p)}^2}$. The vector $(r_Y, r_{Y(p)})$ has an asymptotically normal distribution with mean $(\rho_Y, \rho_{Y(p)})$ and variance

$$\mathbf{\Phi} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}$$

which is given below.

According to the delta method, the expected value of the function $f(r_Y, r_{Y(p)})$ would be the function applied to the expected value of $(r_Y, r_{Y(P)})$, which in large samples is $\text{E}(r_Y, r_{Y(p)}) = (\rho_Y, \rho_{Y(p)})$. So the expected value of $r_{\text{sp}} = f(r_Y, r_{Y(p)}) = \text{sgn}(b_p)\sqrt{r_Y^2 - r_{Y(p)}^2}$ would be $\rho_{sp} = f(\rho_Y, \rho_{Y(p)}) = \text{sgn}(\beta_p)\sqrt{\rho_Y^2 - \rho_{Y(p)}^2}$. Second, the variance of the function of interest can be expressed as

$$\text{var}_\infty(f(r_Y, r_{Y(p)})) = \mathbf{a\Phi a}',$$

where the vector **a** contains the partial derivatives of the function *f* with respect to each of the correlations, and **Φ** is the 2 × 2 variance-covariance matrix for r_Y and $r_{Y(p)}$. From Olkin and Siotani (1976) and Olkin and Finn (1995)

$$\phi_{11} = \text{var}_\infty(r_Y) = (1 - \rho_Y^2)^2 / n,$$

$$\phi_{22} = \text{var}_\infty(r_{Y(p)}) = (1 - \rho_{Y(p)}^2)^2 / n, \text{ and}$$

$$\phi_{12} = \phi_{21} = \text{cov}_\infty(r_Y, r_{Y(p)}) = \left[\frac{1}{2}(2\rho_* - \rho_Y \rho_{Y(p)})(1 - \rho_Y^2 - \rho_{Y(p)}^2 - \rho_*^2) + \rho_*^3\right] / n \quad \text{where}$$

$\rho_* = \dfrac{\rho_{Y(p)}}{\rho_Y}$, as derived by Alf and Graf (1999).

The two partial derivatives of *f* with respect to r_Y and $r_{Y(p)}$ evaluated at the values $(\rho_Y, \rho_{Y(p)})$ in the row vector **a** are

$$a_1 = \frac{\partial}{\partial r_Y}(\sqrt{r_Y^2 - r_{Y(p)}^2})\ \Big|_{\rho_Y, \rho_{Y(p)}} = \frac{1}{2}(\rho_Y^2 - \rho_{Y(p)}^2)^{-1/2}(2\rho_Y)$$
$$= \rho_Y / \sqrt{\rho_Y^2 - \rho_{Y(p)}^2},$$

$$a_2 = \frac{\partial}{\partial r_{Y(p)}}(\sqrt{r_Y^2 - r_{Y(P)}^2})\ \Big|_{\rho_Y^2, \rho_{Y(p)}^2} = \frac{1}{2}(\rho_Y^2 - \rho_{Y(p)}^2)^{-1/2}(-2\rho_{Y(p)})$$
$$= -\rho_{Y(p)} / \sqrt{\rho_Y^2 - \rho_{Y(p)}^2}.$$

Thus, we can write var(r_{sp}) = $\mathbf{a\Phi a'}$ as

$$\mathbf{a\Phi a'} = \left[\frac{\rho_Y}{\sqrt{\rho_Y^2 - \rho_{Y(p)}^2}} \quad \frac{-\rho_{Y(p)}}{\sqrt{\rho_Y^2 - \rho_{Y(p)}^2}}\right]\begin{bmatrix}\phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22}\end{bmatrix}\begin{bmatrix}\rho_Y / \sqrt{\rho_Y^2 - \rho_{Y(p)}^2} \\ -\rho_{Y(p)} / \sqrt{\rho_Y^2 - \rho_{Y(p)}^2}\end{bmatrix}.$$

Making the appropriate cancellations,

$$\text{var}(r_{sp}) = \frac{1}{\left(\rho_Y^2 - \rho_{Y(p)}^2\right)}\left[\rho_Y^2\phi_{11} + \rho_{Y(p)}^2\phi_{22} - 2\rho_Y\rho_{Y(p)}\phi_{12}\right].$$

Finally, substituting for ϕ_{11}, ϕ_{22}, and ϕ_{12} I obtain

$$\text{var}(r_{sp}) = \frac{1}{n\left(\rho_Y^2 - \rho_{Y(p)}^2\right)}\Big[\rho_Y^2(1-\rho_Y^2)^2 + \rho_{Y(p)}^2(1-\rho_{Y(p)}^2)^2$$
$$-2\rho_Y\rho_{Y(p)}[\frac{1}{2}(2\rho_* - \rho_Y\rho_{Y(p)})(1-\rho_Y^2 - \rho_{Y(p)}^2 - \rho_*^2) + \rho_*^3]\Big]. \tag{3.3}$$

With some tedious algebraic manipulation it is possible to re-write formula 3.3 as

$$\text{var}(r_{sp}) = \frac{\rho_Y^4 - 2\rho_Y^2 + \rho_{Y(p)}^2 + 1 - \rho_{Y(p)}^4}{n}. \tag{3.4}$$

With this result, we can say that for large samples, r_{sp} has an asymptotically normal distribution with mean ρ_{sp} and variance var(r_{sp}) given in equation 3.3.

Typically formulas 3.3 or 3.4 would be calculated by substituting sample analogues for the population values. The major shortcoming of these estimators for the variance of r_{sp} is that some of the components needed to compute formulas 3.3 are 3.4 are often not reported in primary studies. Specifically, while the R^2 for the full model (r_Y^2) is often presented, typically $r_{Y(p)}^2$ (which appears in 3.3 and 3.4 and is also used to estimate ρ_*) is often not reported. However, thanks to the relationship between the semi-partial correlation and the difference between multiple correlations we can compute the missing $r_{Y(p)}^2$ value. Beginning with formula 3.1, $r_{sp} = \text{sgn}(b_p)\sqrt{r_Y^2 - r_{Y(p)}^2}$, I can solve for $r_{Y(p)}^2$. By so doing I obtain

$$r_{Y(p)}^2 = r_Y^2 - r_{sp}^2 .$$

This may seem circular, but since r_{sp} is typically computed from formula 3.2, it is computed without explicit knowledge of $r_{Y(p)}^2$.

Linear Models with Two Predictors

In this section I examine the general conditions under which one can obtain either r or r_{sp} from multiple regression results. I consider the simplest case, where only two predictors are in the model. However, the ideas presented here can be generalized to p-predictor models.

The general linear model with two predictors can be written as

$$Y_j = \beta_0 + \beta_1 X_{1j} + \beta_2 X_{2j} + e_j$$

where Y_j is the value of the dependent variable for the jth case, X_{mj} is the value of the mth of $p = 2$ predictors for case j, β_0 is the intercept, β_m is the mth slope parameter, and e_j is the random error for the jth observation. The usual assumptions of constant error variance, normality, and independence apply. Suppose one is interested in the relationship between X_2 and Y. As above, let r_{Y1}, r_{Y2}, and r_{12} be the sample correlations between Y and X_1, Y and X_2, and X_1 and X_2, respectively.

X_1 and X_2 are uncorrelated. When $r_{12} = 0$, the slope b_2 from the two-predictor model will be identical to the slope for the bivariate (one-predictor) regression model. That is, $b_1 = r_{Y1}\, S_Y / S_1$, where S_Y and S_1 are the standard deviations for Y and X_1,

respectively. In such cases, one can compute the zero-order r directly from the t test of the regression slope for X_1.

To test the hypothesis that the slope for X_1 is zero in the population, a ratio of b_1 to its standard error (SE) is created and the result is compared to the t critical value with $n - p - 1$ (here, $n - 3$) degrees of freedom. The standard error of b_1 (e.g., Cohen et al., 2003) is

$$SE(b_1) = \frac{S_Y \sqrt{1 - r_Y^2}}{S_1 \sqrt{1 - r_{12}^2} \sqrt{n - p - 1}}, \qquad (3.5)$$

where S_Y is the standard deviation of Y, S_1 is the standard deviation of X_1, and r_Y^2 is the squared multiple correlation for the full equation. With simple algebra it is possible to show that

$$t_1 = \frac{b_1}{SE(b_1)} = \frac{r_{Y1} \dfrac{S_Y}{S_1}}{\dfrac{S_Y \sqrt{1 - r_Y^2}}{S_1 \sqrt{1 - r_{12}^2} \sqrt{n - p - 1}}} = \frac{r_{Y1} \sqrt{1 - r_{12}^2} \sqrt{n - p - 1}}{\sqrt{1 - r_Y^2}}.$$

However, because $r_{12} = 0$ the numerator simplifies, and one can rewrite the equation as

$$t_1 = \frac{r_{Y1} \sqrt{n - p - 1}}{\sqrt{1 - r_Y^2}}. \qquad (3.6)$$

Thus, it is possible to compute the zero-order r via $r_{Y1} = t_1 \sqrt{1 - r_Y^2} \big/ \sqrt{n - p - 1}$ if the R^2 for the full equation is available. Similarly if we substitute t_1 shown in 3.6 into the formula for r_{sp} given in display 3.2, one can simplify that equation and find that $r_{sp} = r$ when $r_{12} = 0$.

X_1 and X_2 are correlated. If the predictor variables are correlated, that is, when $r_{12} \neq 0$, the t statistic for the test that the slope $\beta_2 = 0$ is

$$t_{1C} = \left(\frac{r_{Y1} - r_{Y2} r_{12}}{\sqrt{1 - r_{12}^2}} \right) \frac{\sqrt{(n - p - 1)}}{\sqrt{1 - r_Y^2}}, \qquad (3.7)$$

which holds because

$$b_1 = \left(\frac{r_{Y1} - r_{Y2} r_{12}}{(1 - r_{12}^2)} \right) \frac{S_Y}{S_1} \text{ and } SE(b_1) = \frac{S_Y \sqrt{1 - r_Y^2}}{S_1 \sqrt{1 - r_{12}^2} \sqrt{(n - p - 1)}}.$$

It is possible to solve for r_{Y1} and obtain

$$r_{Y1} = t_{1C} \frac{\sqrt{1 - r_Y^2} \sqrt{1 - r_{12}^2}}{\sqrt{(n - p - 1)}} + r_{Y2} r_{12}.$$

However, this transformation of t_{1C} to r_{Y1} cannot be applied unless the values of r_{Y2} and r_{12} and the full-model R^2 are also available. Of course if all of these values were reported it is likely that r_{Y1} would also be reported, making the use of a transformation only possible if it is not needed.

Finally, the slope t test t_{2C} from display 3.7 can be substituted into the formula for r_{sp} given in 3.2. This simple substitution leads to cancellations and yet another common form of the semi-partial correlation:

$$r_{sp} = \frac{t_{1C} \sqrt{1 - r_Y^2}}{\sqrt{n - p - 1}} = \left[\left(\frac{r_{Y1} - r_{Y2} r_{12}}{\sqrt{1 - r_{12}^2}} \right) \frac{\sqrt{(n - p - 1)}}{\sqrt{1 - r_Y^2}} \right] \frac{\sqrt{1 - r_Y^2}}{\sqrt{n - p - 1}} = \left(\frac{r_{Y1} - r_{Y2} r_{12}}{\sqrt{1 - r_{12}^2}} \right).$$

This formula also reveals that when the pairs of variables X_1 and X_2, and X_1 and Y are positively intercorrelated, r_{sp} will often be lower than r_{Y1} because of the subtraction of the $r_{Y2} r_{12}$ term in the numerator. However, this may not hold if a suppressor effect appears (e.g., $r_{12} < 0$ and r_{Y1} and r_{Y2} are both positive).

The Relationships Among Partial and Semi-partial Correlations and *r*

In the section above, it was shown how the partial and semi-partial correlations relate to the bivariate correlation when the two predictors in the model are uncorrelated with each other in the sample. Table 3.1 presents some examples of how the partial and semi-partial correlations relate to the bivariate correlation. Specifically, the correlation between X_1 and X_2 (r_{12}) was set to be 0, .2, .4, .6, and .8 (column one), the correlation between Y and X_1 (r_{Y1}) was set to be 0, .2, and .4 (column two), the correlation between Y and X_2 (r_{Y2}) was set to be 0, .4, and .6 (column three). For all combinations, the values of the partial correlations (r_{p1} and r_{p2} in columns four and six) and semi-partial correlations (r_{sp1} and r_{sp2} in columns five and seven) of X_1 and X_2 with Y were compared to the values of the bivariate correlations r_{Y1} and r_{Y2}. Columns eight and ten present the difference between the zero order correlations and partial correlations (r_{Y1} - r_{p1} and r_{Y2} - r_{p2}, respectively) and columns nine and eleven present the difference between the zero order correlations and semi-partial correlations ($r_{Y1} - r_{sp1}$ and $r_{Y2} - r_{sp2}$, respectively). For all cases r_{Y2} is equal to or larger than r_{Y1}.

Table 3.1. Differences Among Correlations, Partial and Semi-partial Correlations

r_{12}	r_{Y1}	r_{Y2}	r_{p1}	r_{sp1}	r_{p2}	r_{sp2}	$(r_{Y1} - r_{p1})$	$(r_{Y1} - r_{sp1})$	$(r_{Y2} - r_{p2})$	$(r_{Y2} - r_{sp2})$
.00	.00	.00	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.00	.20	.40	.2182	.2000	.4082	.4000	.0182	.0000	.0082	.0000
.00	.40	.60	.5000	.4000	.6547	.6000	.1000	.0000	.0546	.0000
.20	.00	.00	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.20	.20	.40	.1336	.1225	.3750	.3674	-.0664	-.0775	-.0250	-.0326
.20	.40	.60	.3572	.2858	.5791	.5307	-.0428	-.1142	-.0209	-.0693
.40	.00	.00	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.40	.20	.40	.0476	.0436	.3563	.3491	-.1524	-.1564	-.0436	-.0509
.40	.40	.60	.2182	.1746	.5238	.4801	-.1818	-.2254	-.0761	-.1199
.60	.00	.00	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.60	.20	.40	-.0546	-.0500	.3572	.3500	-.2546	-.2500	-.0427	-.0500
.60	.40	.60	.0625	.0500	.4910	.4500	-.3375	-.3500	-.1090	-.1500
.80	.00	.00	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.80	.20	.40	-.2182	-.2000	.4082	.4000	-.4182	-.4000	.0082	.0000
.80	.40	.60	-.1667	-.1333	.5092	.4667	-.5667	-.5333	-.0908	-.1333

Table 3.1 shows that when $r_{12} = 0$ only the semi-partial correlation is equal to the bivariate correlation results (r_{Y1} and r_{Y2}). When the predictors are correlated ($r_{12} \neq 0$), the difference between the partial and semi-partial correlations and the bivariate correlation increases as r_{12} increases. Because in Table 3.1 the correlation between *Y* and X_2 (r_{Y2}) is larger than the correlation between *Y* and X_1 (r_{Y1}), the difference between the partial or semi-partial correlation and the bivariate correlation is larger with respect to r_{Y1} than r_{Y2}. For the correlation between *Y* and X_1 (r_{Y1}), when the correlation between X_1 and X_2 (r_{12}) is

larger than the correlation between the correlation between Y and X_1 (r_{Y1}) differences both the partial and the semi-partial correlations are negative even though the correlation between Y and X_1 (r_{Y1}) is positive.

The next step is to examine what value results if one applies the standard $t - \text{to} - r$ transformation shown in formula 3.8 below to t_S, the t test for the slope in the multiple regression given in formula 3.6. Specifically, we examine the case with two predictors in the model. The typical transformation can be written as

$$r_S = \frac{t_S}{\sqrt{t_S^2 + (n-p-1)}}. \tag{3.8}$$

First, we substitute in formula 3.8 the value of the t test from a linear regression with two predictors when $r_{12} = 0$, formula 3.6. If we square both sides of the equation we get

$$r_S^2 = \left(\frac{t_S}{\sqrt{t_S^2 + (n-p-1)}} \right)^2 = \frac{\dfrac{r_{Y1}^2 (n-p-1)}{(1-r_Y^2)}}{\dfrac{r_{Y1}^2 (n-p-1)}{(1-r_Y^2)} + (n-p-1)}$$

$$= \frac{\dfrac{r_{Y1}^2 (n-p-1)}{(1-r_Y^2)}}{\dfrac{r_{Y1}^2 (n-p-1) + (n-p-1)(1-r_Y^2)}{(1-r_Y^2)}}$$

$$= \frac{r_{Y1}^2 (1-r_{Y.12}^2)}{(1-r_Y^2)\left[r_{Y1}^2 + (1-r_Y^2)\right]}$$

$$= \frac{r_{Y1}^2}{\left[r_{Y1}^2 + (1-r_Y^2)\right]}.$$

Rearranging the terms and taking the square root, we obtain

$$r_S = \frac{r_{Y1}}{\sqrt{(1 + r_{Y1}^2 - r_Y^2)}} . \tag{3.9}$$

Examination of formula 3.9 indicates that the closer that r_Y^2 is to r_{Y1}^2, the closer r_S is to r_{Y1}. In the hypothetical case where $r_Y^2 = r_{Y1}^2$, the denominator would be one and r_S would equal r_{Y1}. As we will see below the complexity is greater when X_1 and X_2 are correlated in the sample.

Next I consider the situation in which $r_{12} \neq 0$, in other words the case where the variables X_1 and X_2 are correlated. When the predictors are correlated, the amount of variance in each that is shared with the outcome is affected by the magnitudes and signs of their intercorrelation. In addition, the inclusion or exclusion of predictors may alter the relationships of each of the predictors with the outcome. When independent variables are intercorrelated, it is not possible to state with certainty what proportion of variance in the dependent variable is explained by each of the independent variables.

In this situation, following the steps shown above, the slope is

$$b_{SC} = \left(\frac{r_{Y1} - r_{Y2} r_{12}}{(1 - r_{12}^2)} \right) \frac{S_{1.2}}{S_1} , \tag{3.10}$$

where $S_{1.2}$ is the standard error of X_1 partialing out X_2. Following similar procedures to those used above, the t test formula when $r_{12} \neq 0$ can be expressed as follows.

$$t_{SC} = \frac{\left(\frac{r_{Y1} - r_{Y2} r_{12}}{(1 - r_{12}^2)} \right) \frac{S_Y}{S_1}}{\frac{S_{Y.1}}{S_{1.2} \sqrt{n-1}}} = \frac{\left(\frac{r_{Y1} - r_{Y2} r_{12}}{(1 - r_{12}^2)} \right) S_Y S_{1.2} \sqrt{(n-1)}}{S_1 S_Y \sqrt{1 - r_Y^2} \frac{\sqrt{(n-1)}}{\sqrt{(n-p-1)}}}$$

$$= \frac{\left(\frac{r_{Y1} - r_{Y2} r_{12}}{(1 - r_{12}^2)} \right) S_{1.2} \sqrt{(n-p-1)}}{S_1 \sqrt{1 - r_Y^2}} ,$$

$$t_{SC} = \left(\frac{r_{Y1} - r_{Y2} r_{12}}{(1 - r_{12}^2)} \right) \frac{S_{1.2}}{S_1} \frac{\sqrt{(n-p-1)}}{\sqrt{1 - r_Y^2}} . \tag{3.11}$$

Here $S_{1.2} \neq S_1$ so these terms don't cancel out. Next, replacing t_{SC} in formula 3.8 and following the same steps used above, when $r_{12} \neq 0$ we obtain the following

$$r_{SC}^2 = \frac{\left[\left(\frac{r_{Y1} - r_{Y2} r_{12}}{(1 - r_{12}^2)} \right) \frac{S_{1.2}}{S_1} \frac{\sqrt{(n-p-1)}}{\sqrt{1 - r_Y^2}} \right]^2}{\left[\left(\frac{r_{Y1} - r_{Y2} r_{12}}{(1 - r_{12}^2)} \right) \frac{S_{1.2}}{S_1} \frac{\sqrt{(n-p-1)}}{\sqrt{1 - r_Y^2}} \right]^2 + (n-p-1)} .$$

Let $A = \frac{r_{Y1} - r_{Y2} r_{12}}{(1 - r_{12}^2)}$, then

$$r_{SC}^2 = \frac{A^2 \left(\frac{S_{1.2}}{S_1} \right)^2 \frac{(n-p-1}{(1 - r_Y^2)})}{A^2 \left(\frac{S_{1.2}}{S_1} \right)^2 \frac{(n-p-1)}{(1 - r_Y^2)} + (n-p-1)} = \frac{A^2 \left(\frac{S_{1.2}}{S_1} \right)^2 \frac{(n-p-1)}{(1 - r_Y^2)}}{\frac{A^2 S_{1.2}^2 (n-p-1) + S_1^2 (n-p-1)}{S_1^2 (1 - r_Y^2)}}$$

$$= \frac{A^2 \left(\frac{S_{1.2}}{S_1} \right)^2 \frac{(n-p-1)}{(1 - r_Y^2)} S_1^2 (1 - r_Y^2)}{(n-p-1)[A^2 S_{1.2}^2 + S_1^2 (1 - r_Y^2)]} = \frac{A^2 S_{1.2}^2 (n-p-1)}{A^2 S_{1.2}^2 + S_1^2 (1 - r_Y^2)} .$$

Also,

$$r_{SC}^2 = \frac{A^2 S_{1.2}^2}{A^2 S_{1.2}^2 + S_1^2 (1 - r_Y^2)} \Rightarrow r_{SC} = \frac{A S_{1.2}}{\sqrt{A^2 S_{1.2}^2 + S_1^2 (1 - r_Y^2)}}$$

thus,

$$r_{SC} = \frac{\frac{r_{Y1} - r_{Y2} r_{12}}{(1 - r_{12}^2)} S_{1.2}}{\sqrt{\left(\frac{r_{Y1} - r_{Y2} r_{12}}{(1 - r_{12}^2)} \right)^2 S_{1.2}^2 + S_1^2 (1 - r_Y^2)}} . \tag{3.12}$$

Formula 3.12 is not equal to formula 3.2, the equation for the semi-partial correlation.

Finally, Table 3.2 presents some examples of how the partial and semi-partial correlations relate to the standardized slope. Specifically, the correlation between X_1 and X_2 (r_{12}) was set to be 0, .2, .4, .6, and .8 (column one), the correlation between Y and X_1 (r_{Y1}) was set to be 0, .2, and .4, (column two), and the correlation between Y and X_2 (r_{Y2}) was set to be 0, .4, and .6 (column three). For all combinations, the values of the standardized slope for X_1 (b_1*, column four), the partial correlation for X_1 (column five) and semi-partial correlation for X_1 (column six) were computed. Columns seven and eight present the difference between the standardized slope and partial correlations ($b_1^* - r_{p1}$) and the standardized slope and semi-partial correlation ($b_1^* - r_{sp1}$), respectively. For all cases r_{Y2} is equal to or larger than r_{Y1}.

Table 3.2 shows that when $r_{12} = 0$ only the semi-partial correlation is equal to the standardized slope. When the predictors are correlated ($r_{12} \neq 0$), the difference between the partial and semi-partial correlations and the standardized slope increases as r_{12} increases. When the correlation between Y and X_2 (r_{Y2}) is larger than the correlation between Y and X_1 (r_{Y1}) the partial correlation differed less than the semi-partial correlation from the slope.

Table 3.2. Differences Between Partial, Semi-partial Correlations, and Slopes

r_{12}	r_{Y1}	r_{Y2}	b_1^*	r_{p1}	r_{sp1}	$(b_1^* - r_{p1})$	$(b_1^* - r_{sp1})$
0.0	0.0	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.0	0.2	0.4	0.2000	0.2182	0.2000	-0.0182	0.0000
0.0	0.4	0.6	0.4000	0.5000	0.4000	-0.1000	0.0000
0.2	0.0	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.2	0.2	0.4	0.1250	0.1336	0.1225	-0.0086	0.0025
0.2	0.4	0.6	0.2917	0.3572	0.2858	-0.0656	0.0059
0.4	0.0	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.4	0.2	0.4	0.0476	0.0476	0.0436	0.0000	0.0040
0.4	0.4	0.6	0.1905	0.2182	0.1746	-0.0277	0.0159
0.6	0.0	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.6	0.2	0.4	-0.0625	-0.0546	-0.0500	-0.0079	-0.0125
0.6	0.4	0.6	0.0625	0.0625	0.0500	0.0000	0.0125
0.8	0.0	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.8	0.2	0.4	-0.3333	-0.2182	-0.2000	-0.1151	-0.1333
0.8	0.4	0.6	-0.2222	-0.1667	-0.1333	-0.0556	-0.0889

CHAPTER 4

EXAMPLE

Data for this example are from Aloe and Becker's (2008) meta-analysis of the relationship between teachers' verbal ability and school outcomes. Table 4.1 lists the results reported in the studies used to compute the r_{sp} values for a subset of k = 13 effects.

Table 4.1. The r_{sp} values

Study	Sample Size	Number of predictors	R^2	t	r_{sp}
1	799	5	0.448	3.300	0.00309
2	524	5	0.584	2.500	0.00311
3	889	23	0.550	-0.380	-0.00029
4	888	23	0.840	6.030	0.00279
5	885	23	0.850	6.100	0.00274
6	884	23	0.880	5.820	0.00234
7	856	24	0.880	5.600	0.00233
8	856	24	0.890	6.800	0.00271
9	595	19	0.530	1.610	0.00192
10	205	7	0.620	1.341	0.00420
11	207	8	0.660	-0.576	-0.00170
12	199	8	0.540	-0.472	-0.00168
13	203	7	0.630	-4.953	-0.01545

As an example below I demonstrate the computations for the r_{sp} index for study 1. In a primary study the general linear model can be expressed as

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \ldots + \beta_p X_{ip} + e_i$$

where Y_i is the value of the dependent variable (e.g., students' academic achievement) for the ith individual, X_{ij} is the value of the jth of p quantitative explanatory variables (e.g., X_{i1} is teachers' verbal ability), β_0 is the intercept, β_j is the jth slope parameter to be estimated from the data, and e_i is the random error for the ith observation. The usual assumptions of constant error variance, normality, and independence apply. Suppose one is interested in the relationship between X_1 and Y. Thus, the semi-partial correlation (r_{sp}) can be computed using formula 3.2 as follows

$$r_{\text{sp}} = \frac{t_p\sqrt{(1-r_Y^2)}}{\sqrt{(n-P-1)}} = \frac{3.3\sqrt{(1-.448)}}{\sqrt{(799-5-1)}} = .00309.$$

Next the variance for each effect can be computed using formula 3.14. Table 4.2 presents the variance for each effect size as well as the components needed to compute the variance.

Table 4.2. The var(r_{sp}) values

r_Y^2	$r_{Y(p)}^2$	r_Y	$r_{Y(p)}$	r_{sp}	r_*	N	var(r_{sp})
0.4480	0.44799	0.66933	0.66932	0.00309	0.99999	799	0.00069
0.5840	0.58399	0.76420	0.76419	0.00311	0.99999	514	0.00081
0.5500	0.55000	0.74162	0.74162	-0.00029	1.00000	889	0.00051
0.8400	0.83999	0.91652	0.91651	0.00279	1.00000	888	0.00018
0.8500	0.84999	0.92195	0.92195	0.00274	1.00000	885	0.00017
0.8800	0.87999	0.93808	0.93808	0.00234	1.00000	884	0.00014
0.8800	0.87999	0.93808	0.93808	0.00233	1.00000	856	0.00014
0.8900	0.88999	0.94340	0.94339	0.00271	1.00000	856	0.00013
0.5300	0.53000	0.72801	0.72801	0.00192	1.00000	595	0.00079
0.6200	0.61998	0.78740	0.78739	0.00420	0.99999	205	0.00185
0.6600	0.66000	0.81240	0.81240	-0.00170	1.00000	207	0.00164
0.5400	0.54000	0.73485	0.73484	-0.00168	1.00000	199	0.00231
0.6300	0.62976	0.79373	0.79358	-0.01545	0.99981	203	0.00182

Typical meta-analysis techniques can now be applied to this data set to estimate an overall weighted effect. Similarly, one can test for the homogeneity of the effects using the Q statistic (Hedges & Olkin, 1985). Let the symbol T_i be a semi-partial correlation ($T_i = r_{\text{sp}i}$). If we denote our estimates of effect size from each of k studies as $\mathbf{T} = (T_i, \ldots, T_k)'$ one can test whether the effect-size estimates are homogeneous, i.e., are estimating a common population effect size θ, using the statistic

$$Q = \sum_{i=1}^{k} \frac{\left(T_i - \bar{T}_\bullet\right)^2}{v_i},$$

where $\overline{T}_{\bullet}$ is the inverse-variance weighted mean effect size and v_i is the within-study sampling variance for the effect size from study *i*, which for most effect-size measures is a function of the sample size. Here v_i is var(r_{sp}) for study *i*, so for this example formula 3.3 can be used to compute the variance. The weighted mean effect size across all *k* studies is given by

$$\overline{T}_{\bullet} = \frac{\sum_{i=1}^{k} \frac{T_i}{v_i}}{\sum_{i=1}^{k} \frac{1}{v_i}}.$$

If Q exceeds the critical value of the χ^2 distribution with k -1 degrees of freedom, the null hypothesis H_0: $\theta_1 = \theta_2 = \ldots = \theta_k = \theta$ is rejected. Thus, a statistically significant Q indicates that the effect-size estimates from the *k* studies do not share the same population effect size.

In this data set no heterogeneity is present among the 13 effect sizes, with Q_T = 0.21, p = 1.00. The weighted overall mean under the fixed-effects model for the 13 semi-partial correlations is .002 (*SE* = 0.0049, z = .43, p = .664), with a 95% *CI* from -.009 to .0128 (see Figure 4.1). This indicates that teachers' verbal ability explains no significant variability in school outcomes beyond the other predictors already included in these 13 equations.

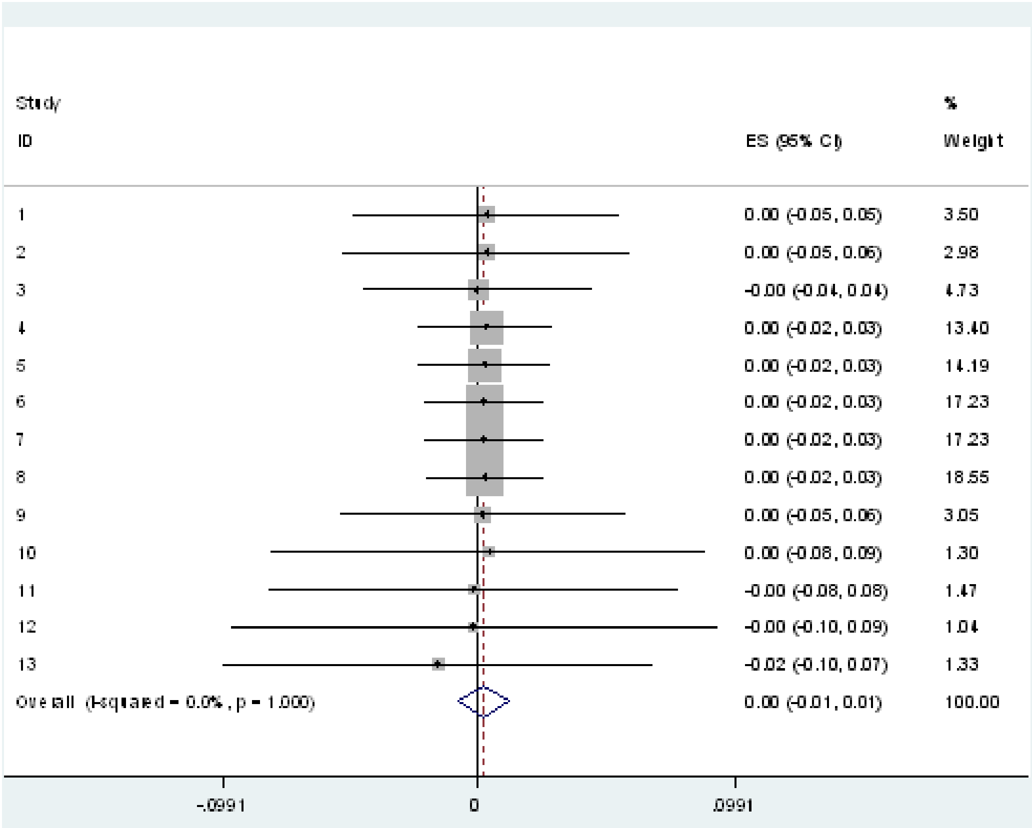

Figure 4.1. Confidence Intervals for Example Data

CHAPTER 5

SIMULATION

In order to evaluate the results presented above, which are based on large-sample theory, I conducted a simulation study. The simulation study was programmed in SAS 9.1. The Cholesky decomposition was used to obtain the desired correlations among the predictors and outcome. For simulating each individual study r_{sp} value, I consider multiple regression models with two, three, five, and ten continuous predictors and a continuous outcome.

In each condition 5,000 replications are generated and the sample size is set to n = 50, 100, 200, 400, or 800. For each replication r_{sp} and var(r_{sp}) were calculated using formulas 3.2 and 3.3 respectively (see simulation code in Appendix B). Specifically, Table 5.1 and 5.2 present the list of values used for the combinations of correlations and intercorrelations.

Data Generation

The case with two predictors. To simulate the data for each pseudo study, a multivariate normal distribution,

$$\left[N\left(\begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_2^2 & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_3^2 \end{pmatrix} \right) \right],$$

is generated, with means $\mu_j = 0.0$, and variances $\sigma_j^2 = 1$. By manipulating the covariances σ_{st}, different levels of collinearity and relatedness of the predictors to the outcome are obtained. A random sample of size n is generated for each multivariate normal distribution. From each pseudo sample, the estimated regression slopes, correlations between predictors, and correlations for each predictor with the outcome are obtained.

Table 5.1. Population Correlations for Two Independent Variables

Conditions			
Number	ρ_{Y1}	ρ_{Y2}	ρ_{12}
1	0.2	0.2	0.0
2	0.2	0.2	0.2
3	0.2	0.2	0.4
4	0.2	0.4	0.0
5	0.2	0.4	0.2
6	0.2	0.4	0.4
7	0.2	0.6	0.0
8	0.2	0.6	0.2
9	0.2	0.6	0.4
10	0.4	0.2	0.2
11	0.4	0.2	0.4
12	0.4	0.2	0.6
13	0.4	0.4	0.2
14	0.4	0.4	0.4
15	0.4	0.4	0.6
16	0.4	0.6	0.2
17	0.4	0.6	0.4
18	0.4	0.6	0.6
19	0.6	0.2	0.0
20	0.6	0.2	0.2
21	0.6	0.2	0.4
22	0.6	0.4	0.2
23	0.6	0.4	0.4
24	0.6	0.4	0.6
25	0.6	0.6	0.2
26	0.6	0.6	0.4
27	0.6	0.6	0.6

The case with three predictors. To simulate the data for each pseudo study a multivariate normal distribution,

$$\left[N\left(\begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} & \sigma_{14} \\ \sigma_{21} & \sigma_2^2 & \sigma_{23} & \sigma_{24} \\ \sigma_{31} & \sigma_{32} & \sigma_3^2 & \sigma_{34} \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_4^2 \end{pmatrix} \right) \right],$$

was generated, with means $\mu_j = 0.0$, and variances $\sigma_j^2 = 1$. By manipulating the covariances σ_{st} different levels of collinearity were obtained. A random sample of size *n* was generated for each multivariate normal distribution. From each pseudo sample, the estimated regression slopes, correlations between predictors, and correlations for each predictor with the outcome is obtained.

Table 5.2. Population Correlations for Three Independent Variables

Conditions				
Number	ρ_{Y1}	ρ_{12}	ρ_{13}	ρ_{23}
1	0.2	0.0	0.1	0.2
2	0.2	0.0	0.2	0.2
3	0.2	0.1	0.1	0.1
4	0.2	0.2	0.2	0.2
5	0.2	0.4	0.1	0.1
6	0.2	0.4	0.2	0.2
7	0.4	0.0	0.1	0.2
8	0.4	0.0	0.2	0.2
9	0.4	0.1	0.1	0.1
10	0.4	0.2	0.2	0.2
11	0.4	0.4	0.1	0.1
12	0.4	0.4	0.2	0.2

The cases with five and ten predictors. Similar procedures to the ones described for the cases with two and three independent variables were utilized for the cases with five and ten predictors.

Parameters

The case with two predictors. The following factors are manipulated in the simulation (see Table 5.1):

(a) The correlation between the predictors is manipulated through the correlation matrix. The values are $\rho_{12} = .0, .2$, and .4.

(b) The correlations between the predictors and the outcome are manipulated, taking on the values ρ_{Y1} = .0, .2, .4, and ρ_{Y2} = .0, .4, and .6.

The 27 conditions for the two independent variables model are specified in Table 5.1.

The case with three predictors. The following factors are manipulated in the simulation (see Table 5.2):

(a) The correlations between the predictors are manipulated through the correlation matrix. The values will be ρ_{12} = .0, .1, .2, and .4, ρ_{13} = .1 and .2, ρ_{23} = .1 and .2.

(b) The correlations between the predictors and with the outcome are manipulated, taking on the values ρ_{Y1} = .2 and .4, and ρ_{Y2} = ρ_{Y3} = .2.

Specifically, the 12 conditions for the three independent variables model are specified in Table 5.2.

The cases with five and ten predictors. For the models with five and ten independent variables only one condition was generated. Specifically, all the correlations among the predictors and between the predictors and the dependent variable for these two models were .2.

Data Analysis

The means of r_{sp} and its variance var(r_{sp}) from 5,000 replications are calculated for each predictor under the conditions specified above. The r_{sp} index and its variance are calculated using formulas 3.2 and 3.3, respectively, then the mean values are compared to the population values that are used to generate the data.

The bias was computed for each condition to quantify the difference between the population value and the estimated value. Specifically, the bias of r_{sp} was computed as

$$Bias(r_{sp}) = \bar{r}_{sp} - \rho_{sp},$$

where ρ_{sp} is the population value and $\bar{r}_{sp}$ is the mean of the estimates in each condition. And the bias of var(r_{sp}) was computed as

$$Bias(\mathrm{var}(r_{sp})) = \overline{\mathrm{var}}(r_{sp}) - \sigma^2(r_{sp}),$$

where $\sigma^2(r_{\text{sp}})$ is the population value and $\overline{\text{var}}(r_{\text{sp}})$ is the mean of the estimates in each condition.

The mean square error (*MSE*) quantifies the overall amount of difference between an estimator and the true value of the quantity being estimated. Specifically, the MSE is calculated as

$$MSE(r_{\text{sp}}) = \text{E}(r_{\text{sp}} - \rho_{\text{sp}})^2 = v(r_{\text{sp}}) + (Bias(r_{\text{sp}}))^2,$$

and

$$MSE(\text{var}(r_{\text{sp}})) = \text{E}(\text{var}(r_{\text{sp}}) - \sigma^2(r_{\text{sp}}))^2 = v(\text{var}(r_{\text{sp}})) + (Bias(\text{var}\, r_{\text{sp}})))^2.$$

To study the impacts of sample size, and the correlations among variables on the estimates of r_{sp} and var(r_{sp}), a factorial Analysis of Variance (ANOVA) was conducted on the results of each condition. The outcome was the difference between the estimate and true value, which are calculated for the 5,000 replications generated for each condition. The factors for the cases with two and three independent variables were the correlations between the predictors and the outcome and the intercorrelation among the predictors. For the cases with five and ten independent variables the factors were the sample size and the number of predictors.

CHAPTER 6

SIMULATION RESULTS

The simulation results are presented in this chapter. First, I discuss in detail the results (minimum, maximum, mean, and standard deviation) for the case with two independent predictors for condition 1 ($\rho_{Y1} = \rho_{Y2} = .2$ and $\rho_{12} = 0$). Second, I discuss the bias for all conditions for the models with two predictors. Next, I discuss the bias in r_{sp} and its variance for all conditions for the models with three predictor variables. Then, I discuss the results for five and ten predictor variables (where all the correlations among predictors and between the predictor and the dependent variable were .2). In each case analysis of variance was used to examine the simulated r_{sp} and var(r_{sp}) values.

The case with two predictors, condition 1

In this section I discuss the results for condition 1 ($\rho_{Y1} = \rho_{Y2} = .2$ and $\rho_{12} = 0$) in detail. Under this condition the population value ρ_{sp} is .20. Figure 6.1 shows the distributions for r_{sp} and Figure 6.2 shows the distributions of var(r_{sp}) for condition 1. In addition, since the scale of the values of var(r_{sp}) decreases rapidly when the sample size increases, Figure 6.3.A and Figure 6.3.B show the distributions of var(r_{sp}) for $n = 400$ and $n = 800$, respectively.

All the r_{sp} distributions seem fairly normal, and when sample size increases the distributions are narrower. In addition, for the larger sample size ($n = 800$) the mean of var(r_{sp}) is .0011, with $SD = 0.000034$. The precision of the estimate is affected by the sample size. Specifically, the smaller the sample size, the smaller the var(r_{sp}) value. In addition, from Table 6.1 the empirical variances of var(r_{sp}) are slightly larger than the values of var(r_{sp}) obtained using formula 3.3. However, the empirical variances are generally very close to the mean var(r_{sp}) values obtained from the simulation. Because the variance of the r_{sp} shows small sample bias, the difference between these two values decreased when the sample size increases. This is another indication that this index behaves as expected.

Although almost all the bias values are negative, indicating that the estimators of r_{sp} and var(r_{sp}) underestimate the population parameters, all the values are quite small and virtually zero for the larger sample sizes. In addition, the larges percentage bias for both r_{sp} and var(r_{sp}) are 1.7% and 7.7%, respectively. The bias and MSE of both r_{sp} and var(r_{sp}) decrease when sample size increases. The largest MSE values were 0.01770786 and 0.00000562 for r_{sp} and var(r_{sp}) respectively. The smallest MSEs were 0.0011062278 and 0.0000000012 for r_{sp} and var(r_{sp}) respectively. Because the values of the MSEs decrease quite rapidly when the sample size increases (in particular for var(r_{sp})), the decision was made to report only the bias for all other simulated data conditions.

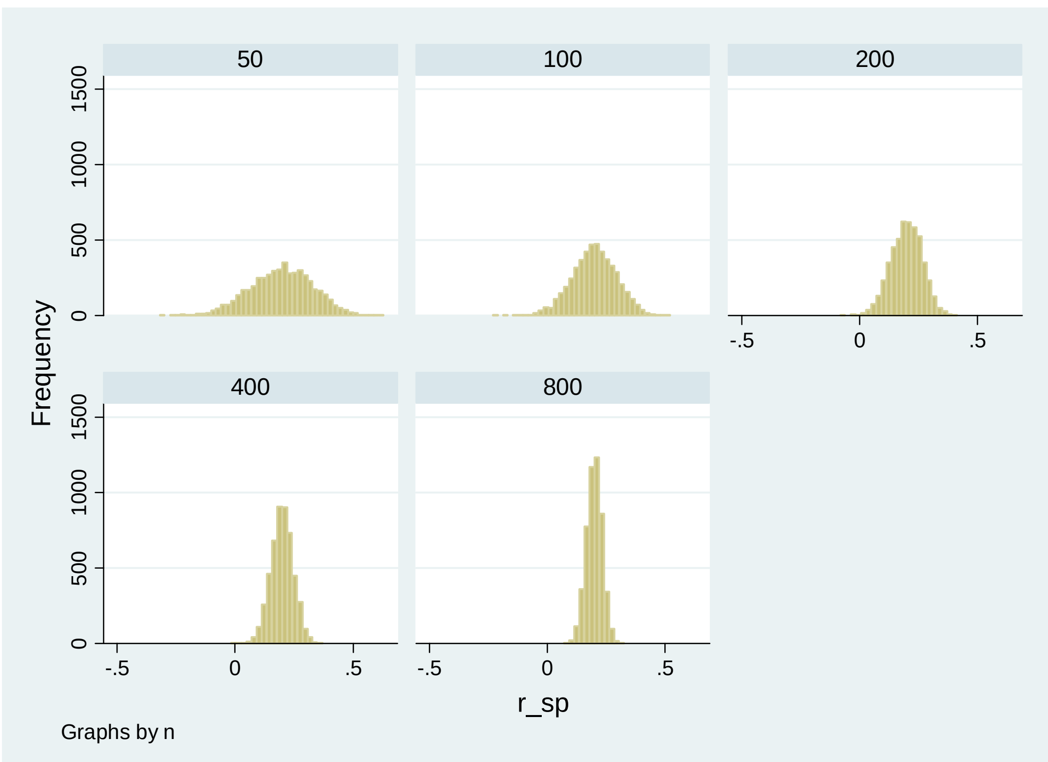

Figure 6.1. Histograms of r_{sp} for condition 1 ($\rho_{Y1} = \rho_{Y2} = .2$ and $\rho_{12} = 0$)

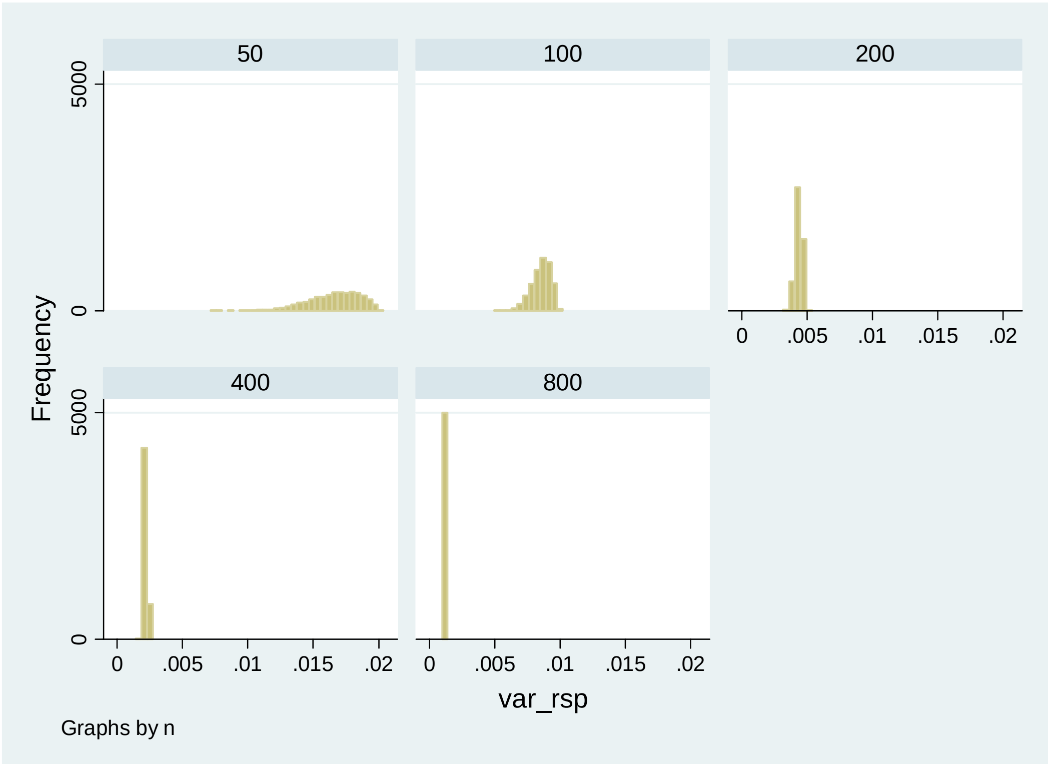

Figure 6.2. Histograms of var(r_{sp}) for condition 1 ($\rho_{Y1} = \rho_{Y2} = .2$ and $\rho_{12} = 0$)

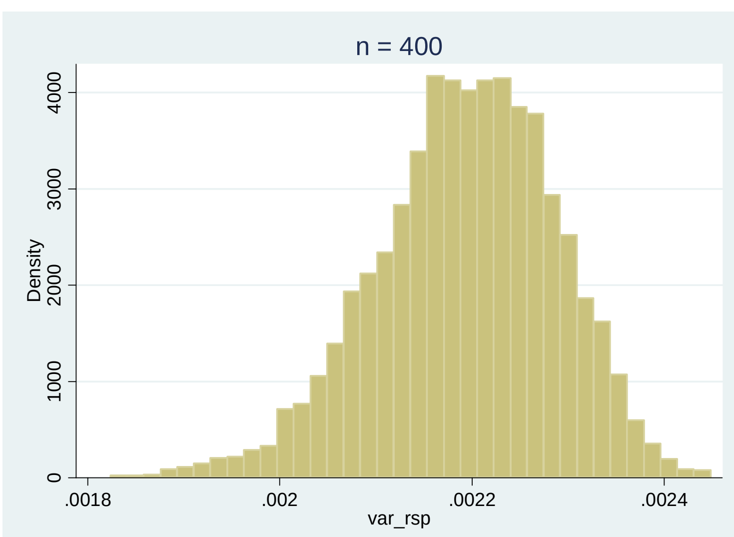

Figure 6.3.A. Histogram of var(r_{sp}) for condition 1 for $n = 400$ ($\rho_{Y1} = \rho_{Y2} = .2$ and $\rho_{12} = 0$)

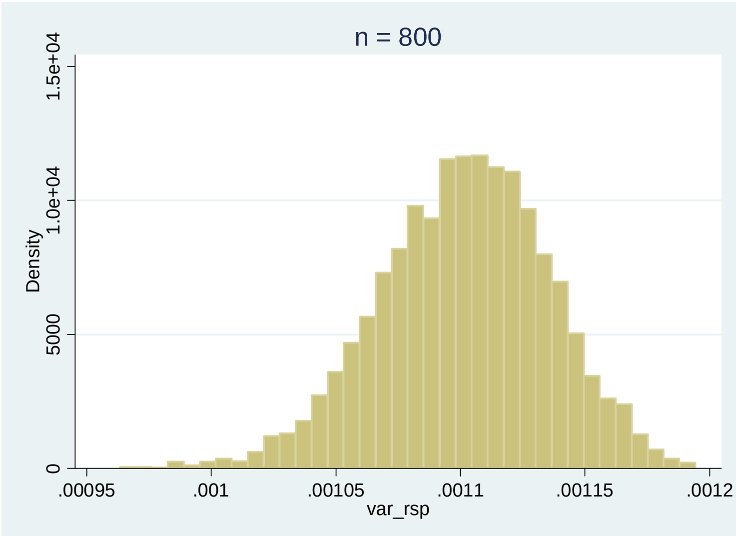

Figure 6.3.B. Histogram of var(r_{sp}) for condition 1 for n = 800 ($\rho_{Y1} = \rho_{Y2} = .2$ and $\rho_{12} = 0$)

Table 6.1. Summary Statistics for Condition 1 ($\rho_{Y1} = \rho_{Y2} = .2$ and $\rho_{12} = 0$)

	Min	Max	Mean	SD	Empirical Variance	Bias	MSE
				n = 50			
r_{sp}	-0.32	0.61	0.19656	0.134766	0.01816	-0.003444	0.017707861
var(r_{sp})	0.007	0.019	0.01651	0.002051		-0.001190	0.000005621
				n = 100			
r_{sp}	-0.22	0.52	0.19901	0.094434	0.00892	-0.000995	0.008848991
var(r_{sp})	0.005	0.009	0.00854	0.000747		-0.000304	0.000000650
				n = 200			
r_{sp}	-0.08	0.41	0.19866	0.066934	0.00448	-0.001345	0.004425809
var(r_{sp})	0.003	0.005	0.00435	0.000265		-0.000071	0.000000075
				n = 400			
r_{sp}	-0.01	0.36	0.19944	0.047412	0.00225	-0.000559	0.002212312
var(r_{sp})	0.001	0.002	0.00219	0.000094		-0.000019	0.000000009
				n = 800			
r_{sp}	0.08	0.32	0.19952	0.033425	0.00112	-0.000477	0.001106228
var(r_{sp})	0.0009	0.001	0.00110	0.000034		-0.000005	0.000000001

The case with two predictors, all conditions

The biases for r_{sp} and var(r_{sp}) are presented in Table 6.2. Figures 6.4 and 6.5 present the biases for both r_{sp} and var(r_{sp}) for all conditions detailed in Chapter 5, Table 5.1. From Table 6.2 it can be seen that the estimators of r_{sp} and var(r_{sp}) underestimate the population values. The population values of r_{sp} and var(r_{sp}) depend on ρ_{Y1}, ρ_{Y2}, and ρ_{12}. Specifically, when ρ_{Y1} and ρ_{Y2} are held constant and ρ_{12} increases, the population value of r_{sp} decreases. The opposite is true for the population value of var(r_{sp}): When ρ_{Y1} and ρ_{Y2} are held constant and ρ_{12} increases, the population value of var(r_{sp}) increases. In addition, Table 6.2 indicates that the empirical variance of r_{sp} is almost always slightly larger than the population value of var(r_{sp}) except for larger sample sizes.

For each individual condition the bias of r_{sp} and var(r_{sp}) decreases when the sample size increases. For instance, for condition 1 when the sample size is 50 the biases for r_{sp} and var(r_{sp}) are -0.00344 and -0.0011898, respectively. Then for the larger sample size (n = 800) the biases for r_{sp} and var(r_{sp}) drop to -0.00048 and -0.0000046, respectively. The same pattern is observed for other conditions, which clearly indicates that when the sample size increases, the biases for both r_{sp} and var(r_{sp}) decrease. For the larger sample size the biases reduce to virtually zero.

Next, I discuss the ANOVA analyses for the model with two independent variables for r_{sp} and var(r_{sp}). The factors for this model are ρ_{Y1}, ρ_{Y2}, and ρ_{12}, and different analyses were run for each sample size to reduce heteroskedasticity. Nevertheless, for all the models the Levene's test of homogeneity of variance was statistically significant at .001 indicating the assumption of homogeneity of variance is not met. A possible explanation is that given the large number of replications the variances for each factor level still differ even though different analyses are performed for different sample sizes.

For n = 50, the results for the differences between r_{sp} and ρ_{sp} indicate that only two (ρ_{Y1} and ρ_{Y2}) of the three factors are statistically significant (Table 6.3). No interaction effect is found among the factors. The adjusted R^2 for this model is virtually zero, which indicates that although the F tests are statistical significant there is no really much explanatory power for the factors or interactions. Thus, it is possible to believe that the differences among factors have not practical importance.

The main effect for ρ_{Y1} yields an F ratio of $F(2, 134992) = 22.66$, $p < .001$, indicating that the mean difference between r_{sp} and ρ_{sp} values is affected by the correlation among the focal predictor and the outcome. The main effect of ρ_{Y2} yields an F ratio of $F(2, 134992) = 8.99$, $p < .001$, indicating that the means are also affected by the correlation between the other predictor in the model and the outcome. The main effect of ρ_{12} yields an F ratio of $F(2, 134992) = 0.49$, $p = .687$, indicating that the mean differences between r_{sp} and ρ_{sp} values is not affected by the intercorrelation between the two predictors in the model. Figures 6.6 and 6.7 present the estimated marginal means of the bias of r_{sp}.

For $n = 50$, the results for var(r_{sp}) indicate that the three main effects (ρ_{Y1}, ρ_{Y2}, and ρ_{12}) are statistically significant (Table 6.4). Two interaction effects are found among the factors (ρ_{Y1} with ρ_{Y2}, and ρ_{Y2} with ρ_{12}). The adjusted R^2 for this model is again virtually zero.

The main effect for ρ_{Y1} yields an F ratio of $F(2, 134982) = 1719.29$, $p < .001$, indicating that the mean differences between var(r_{sp}) values and the population values are affected by the correlation among the focal predictor and the outcome. The bias of var(r_{sp}) increases when ρ_{Y1} increases (see Figure 6.8). The main effect of ρ_{Y2} yields an F ratio of $F(2, 134982) = 51.55$, $p < .001$, indicating that the mean differences between the var(r_{sp}) estimates and the population values are affected by the correlation between the other predictor in the model and the outcome. The main effect of ρ_{12} yielded an F ratio of $F(3, 134982) = 34.88$, $p < .001$, indicating that the means are also affected by the intercorrelation between the two predictors in the model. In addition, the interaction effect between ρ_{Y1} and ρ_{Y2} yields an F ratio of $F(4, 134982) = 172.00$, $p < .001$, indicating that the largest difference among the means by ρ_{Y1} occurs when $\rho_{Y2} = .2$, the difference is smaller for $\rho_{Y2} = .4$, and it is even smaller for $\rho_{Y2} = .6$ (Figure 6.9). The interaction effect between ρ_{Y2} and ρ_{12} yields an F ratio of $F(6, 134982) = 3.14$, $p = .004$, indicating that for $\rho_{Y2} = .2$ the mean drops rapidly, slows down when $\rho_{12} = .2$ and seems to flatten out when ρ_{12} reaches .4 (Figures 6.10).

The ANOVAs for $n = 100$ and $n = 200$ for r_{sp} are virtually the same as that for the sample size $n = 50$ (see Table A.1 and Table A.3, and Figures A.1, A.2 and Figure A.5, A.6, in Appendix A). However for var(r_{sp}) the only interaction effect that is significant for the larger n values is between ρ_{Y1} and ρ_{Y2} (Table A.2 and Table A.4, and Figures A.3 and A.4, A.6, A.7, and A.8 in Appendix A).

For $n = 400$, the results for the $r_{sp} - \rho_{sp}$ differences indicate that none of the three factors (ρ_{Y1}, ρ_{Y2}, and ρ_{12}) are statistically significant (Table A.5 and Figure A.9 and A.10, in Appendix A). No interaction effect is found among the factors and the adjusted R^2 for this model is virtually zero. In addition, for var(r_{sp}) the only interaction effect that is significant is between ρ_{Y1} and ρ_{Y2} (Table A.6 and Figures A.11 and A.12, Appendix A). The results for $n = 800$ are virtually the same as the results for $n = 400$ (Table A.7 and Table A.8, and Figures A.13, A.14, A.15 and A.16 in Appendix A).

Table 6.2. Population Values, Empirical Means, and Biases for r_{sp} and var(r_{sp}) for Two Independent Variables

	Conditions			r_{sp}			var(r_{sp})		Bias	
				Population	Empirical	Empirical	Population	Empirical		
	ρ_{Y1}	ρ_{Y2}	ρ_{12}	Value	Mean	Variance	Value	Mean	r_{sp}	var(r_{sp})
						n = 50				
1	0.2	0.2	0.0	0.20000	0.19656	0.0181619	0.0176960	0.0165062	-0.00344	-0.0011898
2	0.2	0.2	0.2	0.16330	0.16005	0.0187157	0.0181902	0.0169231	-0.00325	-0.0012671
3	0.2	0.2	0.4	0.13093	0.12776	0.0191323	0.0185476	0.0172243	-0.00317	-0.0013233
4	0.2	0.4	0.0	0.20000	0.19691	0.0160446	0.0154880	0.0145411	-0.00309	-0.0009469
5	0.2	0.4	0.2	0.12247	0.12026	0.0168810	0.0163005	0.0152381	-0.00222	-0.0010624
6	0.2	0.4	0.4	0.04364	0.04228	0.0173217	0.0167361	0.0156101	-0.00136	-0.0011259
7	0.2	0.6	0.0	0.20000	0.19752	0.0123999	0.0118080	0.0112215	-0.00248	-0.0005865
8	0.2	0.6	0.2	0.08165	0.08045	0.0132469	0.0126302	0.0119547	-0.00120	-0.0006756
9	0.2	0.6	0.4	-0.04364	-0.04350	0.0133489	0.0127513	0.0120622	0.00014	-0.0006891
10	0.4	0.2	0.2	0.36742	0.36078	0.0151506	0.0143805	0.0137168	-0.00664	-0.0006637
11	0.4	0.2	0.4	0.34915	0.34233	0.0156465	0.0148161	0.0140975	-0.00682	-0.0007186
12	0.4	0.2	0.6	0.35000	0.34265	0.0156872	0.0147961	0.0140932	-0.00735	-0.0007030
13	0.4	0.4	0.2	0.32660	0.32120	0.0141251	0.0134436	0.0127868	-0.00540	-0.0006567
14	0.4	0.4	0.4	0.26186	0.25699	0.0152733	0.0145900	0.0137841	-0.00487	-0.0008059
15	0.4	0.4	0.6	0.20000	0.19555	0.0161615	0.0154880	0.0145608	-0.00445	-0.0009272
16	0.4	0.6	0.2	0.28577	0.28193	0.0114146	0.0108427	0.0103612	-0.00384	-0.0004816
17	0.4	0.6	0.4	0.17457	0.17177	0.0126102	0.0120384	0.0114351	-0.00281	-0.0006033
18	0.4	0.6	0.6	0.05000	0.04841	0.0132798	0.0127361	0.0120554	-0.00159	-0.0006807
19	0.6	0.2	0.0	0.60000	0.59076	0.0088126	0.0079680	0.0081204	-0.00924	0.0001524
20	0.6	0.2	0.2	0.57155	0.56228	0.0096869	0.0087902	0.0088625	-0.00927	0.0000722
21	0.6	0.2	0.4	0.56737	0.55779	0.0098628	0.0089113	0.0089812	-0.00957	0.0000699
22	0.6	0.4	0.2	0.53072	0.52284	0.0096007	0.0089227	0.0087954	-0.00788	-0.0001273
23	0.6	0.4	0.4	0.48008	0.47238	0.0108395	0.0101184	0.0098763	-0.00770	-0.0002421
24	0.6	0.4	0.6	0.45000	0.44220	0.0115777	0.0108161	0.0105077	-0.00780	-0.0003084
25	0.6	0.6	0.2	0.48990	0.48410	0.0082315	0.0078080	0.0075812	-0.00580	-0.0002268

Table 6.2. Continued

	Conditions			r_{sp}			$var(r_{sp})$		Bias	
				Population Value	Empirical Mean	Empirical Variance	Population Value	Empirical Mean		
	ρ_{Y1}	ρ_{Y2}	ρ_{12}						r_{sp}	$var(r_{sp})$
26	0.6	0.6	0.4	0.39279	0.38753	0.0098023	0.0093264	0.0089916	-0.00526	-0.0003348
27	0.6	0.6	0.6	0.30000	0.29531	0.0111517	0.0106580	0.0102083	-0.00469	-0.0004497
						$n = 100$				
1	0.2	0.2	0.0	0.20000	0.19900	0.0089177	0.0088480	0.0085443	-0.00100	-0.0003037
2	0.2	0.2	0.2	0.16330	0.16256	0.0091200	0.0090951	0.0087702	-0.00074	-0.0003249
3	0.2	0.2	0.4	0.13093	0.13031	0.0092791	0.0092738	0.0089335	-0.00062	-0.0003402
4	0.2	0.4	0.0	0.20000	0.19933	0.0079168	0.0077440	0.0075092	-0.00067	-0.0002348
5	0.2	0.4	0.2	0.12247	0.12234	0.0082547	0.0081503	0.0078855	-0.00014	-0.0002648
6	0.2	0.4	0.4	0.04364	0.04400	0.0084006	0.0083680	0.0080873	0.00036	-0.0002807
7	0.2	0.6	0.0	0.20000	0.19976	0.0061310	0.0059040	0.0057659	-0.00024	-0.0001381
8	0.2	0.6	0.2	0.08165	0.08199	0.0064878	0.0063151	0.0061552	0.00034	-0.0001599
9	0.2	0.6	0.4	-0.04364	-0.04270	0.0064726	0.0063757	0.0062128	0.00095	-0.0001628
10	0.4	0.2	0.2	0.36742	0.36430	0.0072449	0.0071903	0.0070254	-0.00312	-0.0001648
11	0.4	0.2	0.4	0.34915	0.34594	0.0074711	0.0074080	0.0072272	-0.00321	-0.0001809
12	0.4	0.2	0.6	0.35000	0.34643	0.0075399	0.0073981	0.0072198	-0.00357	-0.0001783
13	0.4	0.4	0.2	0.32660	0.32441	0.0068235	0.0067218	0.0065603	-0.00219	-0.0001614
14	0.4	0.4	0.4	0.26186	0.25993	0.0073346	0.0072950	0.0070944	-0.00193	-0.0002007
15	0.4	0.4	0.6	0.20000	0.19827	0.0077507	0.0077440	0.0075110	-0.00173	-0.0002330
16	0.4	0.6	0.2	0.28577	0.28465	0.0055735	0.0054214	0.0053074	-0.00112	-0.0001139
17	0.4	0.6	0.4	0.17457	0.17385	0.0060979	0.0060192	0.0058754	-0.00073	-0.0001438
18	0.4	0.6	0.6	0.05000	0.04980	0.0063627	0.0063681	0.0062052	-0.00020	-0.0001628
19	0.6	0.2	0.0	0.60000	0.59514	0.0041170	0.0039840	0.0040327	-0.00486	0.0000487
20	0.6	0.2	0.2	0.57155	0.56664	0.0045155	0.0043951	0.0044229	-0.00491	0.0000277
21	0.6	0.2	0.4	0.56737	0.56220	0.0046166	0.0044557	0.0044823	-0.00517	0.0000266
22	0.6	0.4	0.2	0.53072	0.52701	0.0045819	0.0044614	0.0044328	-0.00371	-0.0000286
23	0.6	0.4	0.4	0.48008	0.47631	0.0051554	0.0050592	0.0050018	-0.00377	-0.0000574
24	0.6	0.4	0.6	0.45000	0.44601	0.0055378	0.0054081	0.0053330	-0.00399	-0.0000750

Table 6.2. Continued

	Conditions			r_{sp}			$var(r_{sp})$		Bias	
				Population Value	Empirical Mean	Empirical Variance	Population Value	Empirical Mean		
	ρ_{Y1}	ρ_{Y2}	ρ_{12}						r_{sp}	$var(r_{sp})$
25	0.6	0.6	0.2	0.48990	0.48782	0.0040283	0.0039040	0.0038489	-0.00208	-0.0000551
26	0.6	0.6	0.4	0.39279	0.39070	0.0047438	0.0046632	0.0045832	-0.00209	-0.0000800
27	0.6	0.6	0.6	0.30000	0.29797	0.0053562	0.0053290	0.0052209	-0.00203	-0.0001081
						n = 200				
1	0.2	0.2	0.0	0.20000	0.19866	0.0044802	0.0044240	0.0043532	-0.00134	-0.0000708
2	0.2	0.2	0.2	0.16330	0.16231	0.0045691	0.0045476	0.0044707	-0.00099	-0.0000768
3	0.2	0.2	0.4	0.13093	0.13029	0.0046225	0.0046369	0.0045556	-0.00064	-0.0000813
4	0.2	0.4	0.0	0.20000	0.19901	0.0038863	0.0038720	0.0038194	-0.00099	-0.0000526
5	0.2	0.4	0.2	0.12247	0.12197	0.0040622	0.0040751	0.0040145	-0.00050	-0.0000606
6	0.2	0.4	0.4	0.04364	0.04364	0.0041405	0.0041840	0.0041191	0.00000	-0.0000650
7	0.2	0.6	0.0	0.20000	0.19945	0.0029371	0.0029520	0.0029228	-0.00055	-0.0000292
8	0.2	0.6	0.2	0.08165	0.08157	0.0031282	0.0031576	0.0031231	-0.00008	-0.0000344
9	0.2	0.6	0.4	-0.04364	-0.04325	0.0031457	0.0031878	0.0031525	0.00040	-0.0000353
10	0.4	0.2	0.2	0.36742	0.36555	0.0036467	0.0035951	0.0035586	-0.00187	-0.0000366
11	0.4	0.2	0.4	0.34915	0.34747	0.0037278	0.0037040	0.0036625	-0.00168	-0.0000415
12	0.4	0.2	0.6	0.35000	0.34842	0.0037023	0.0036990	0.0036570	-0.00158	-0.0000420
13	0.4	0.4	0.2	0.32660	0.32536	0.0033664	0.0033609	0.0033254	-0.00124	-0.0000355
14	0.4	0.4	0.4	0.26186	0.26097	0.0036309	0.0036475	0.0036018	-0.00089	-0.0000457
15	0.4	0.4	0.6	0.20000	0.19946	0.0038338	0.0038720	0.0038179	-0.00054	-0.0000541
16	0.4	0.6	0.2	0.28577	0.28519	0.0026813	0.0027107	0.0026866	-0.00058	-0.0000241
17	0.4	0.6	0.4	0.17457	0.17437	0.0029686	0.0030096	0.0029785	-0.00020	-0.0000311
18	0.4	0.6	0.6	0.05000	0.05023	0.0031332	0.0031840	0.0031484	0.00023	-0.0000356
19	0.6	0.2	0.0	0.60000	0.59743	0.0020572	0.0019920	0.0020073	-0.00257	0.0000153
20	0.6	0.2	0.2	0.57155	0.56910	0.0022521	0.0021976	0.0022071	-0.00245	0.0000096
21	0.6	0.2	0.4	0.56737	0.56497	0.0022673	0.0022278	0.0022362	-0.00239	0.0000083
22	0.6	0.4	0.2	0.53072	0.52901	0.0022366	0.0022307	0.0022257	-0.00171	-0.0000050
23	0.6	0.4	0.4	0.48008	0.47853	0.0025273	0.0025296	0.0025173	-0.00155	-0.0000123

Table 6.2. Continued

	Conditions			r_{sp}			$var(r_{sp})$		Bias	
				Population Value	Empirical Mean	Empirical Variance	Population Value	Empirical Mean		
	ρ_{Y1}	ρ_{Y2}	ρ_{12}						r_{sp}	$var(r_{sp})$
24	0.6	0.4	0.6	0.45000	0.44855	0.0026958	0.0027040	0.0026867	-0.00145	-0.0000173
25	0.6	0.6	0.2	0.48990	0.48902	0.0019188	0.0019520	0.0019401	-0.00088	-0.0000119
26	0.6	0.6	0.4	0.39279	0.39213	0.0022911	0.0023316	0.0023142	-0.00066	-0.0000174
27	0.6	0.6	0.6	0.30000	0.29956	0.0026213	0.0026645	0.0026406	-0.00044	-0.0000239
						$n = 400$				
1	0.2	0.2	0.0	0.20000	0.19944	0.0022479	0.0022120	0.0021935	-0.00056	-0.0000185
2	0.2	0.2	0.2	0.16330	0.16289	0.0023136	0.0022738	0.0022538	-0.00041	-0.0000200
3	0.2	0.2	0.4	0.13093	0.13067	0.0023626	0.0023184	0.0022974	-0.00026	-0.0000211
4	0.2	0.4	0.0	0.20000	0.19945	0.0019827	0.0019360	0.0019219	-0.00055	-0.0000141
5	0.2	0.4	0.2	0.12247	0.12212	0.0020807	0.0020376	0.0020215	-0.00035	-0.0000161
6	0.2	0.4	0.4	0.04364	0.04350	0.0021275	0.0020920	0.0020749	-0.00014	-0.0000172
7	0.2	0.6	0.0	0.20000	0.19952	0.0015186	0.0014760	0.0014678	-0.00048	-0.0000082
8	0.2	0.6	0.2	0.08165	0.08134	0.0016188	0.0015788	0.0015692	-0.00031	-0.0000096
9	0.2	0.6	0.4	-0.04364	-0.04373	0.0016259	0.0015939	0.0015841	-0.00009	-0.0000098
10	0.4	0.2	0.2	0.36742	0.36657	0.0018414	0.0017976	0.0017878	-0.00086	-0.0000098
11	0.4	0.2	0.4	0.34915	0.34841	0.0018927	0.0018520	0.0018409	-0.00074	-0.0000111
12	0.4	0.2	0.6	0.35000	0.34935	0.0018896	0.0018495	0.0018383	-0.00065	-0.0000112
13	0.4	0.4	0.2	0.32660	0.32585	0.0017250	0.0016804	0.0016711	-0.00075	-0.0000094
14	0.4	0.4	0.4	0.26186	0.26129	0.0018575	0.0018238	0.0018118	-0.00057	-0.0000120
15	0.4	0.4	0.6	0.20000	0.19963	0.0019582	0.0019360	0.0019219	-0.00037	-0.0000141
16	0.4	0.6	0.2	0.28577	0.28518	0.0013920	0.0013553	0.0013488	-0.00060	-0.0000066
17	0.4	0.6	0.4	0.17457	0.17415	0.0015334	0.0015048	0.0014964	-0.00042	-0.0000084
18	0.4	0.6	0.6	0.05000	0.04982	0.0016082	0.0015920	0.0015824	-0.00018	-0.0000096
19	0.6	0.2	0.0	0.60000	0.59870	0.0010346	0.0009960	0.0009998	-0.00130	0.0000038
20	0.6	0.2	0.2	0.57155	0.57036	0.0011307	0.0010988	0.0011009	-0.00118	0.0000021
21	0.6	0.2	0.4	0.56737	0.56628	0.0011387	0.0011139	0.0011155	-0.00108	0.0000016
22	0.6	0.4	0.2	0.53072	0.52969	0.0011452	0.0011153	0.0011142	-0.00103	-0.0000012

Table 6.2. Continued

	Conditions			r_{sp}			$\text{var}(r_{sp})$		Bias	
				Population	Empirical	Empirical	Population	Empirical		
	ρ_{Y1}	ρ_{Y2}	ρ_{12}	Value	Mean	Variance	Value	Mean	r_{sp}	$\text{var}(r_{sp})$
23	0.6	0.4	0.4	0.48008	0.47918	0.0012833	0.0012648	0.0012616	-0.00090	-0.0000032
24	0.6	0.4	0.6	0.45000	0.44924	0.0013600	0.0013520	0.0013474	-0.00076	-0.0000046
25	0.6	0.6	0.2	0.48990	0.48913	0.0009995	0.0009760	0.0009729	-0.00077	-0.0000031
26	0.6	0.6	0.4	0.39279	0.39212	0.0011814	0.0011658	0.0011613	-0.00067	-0.0000045
27	0.6	0.6	0.6	0.30000	0.29950	0.0013348	0.0013323	0.0013260	-0.00050	-0.0000063
						n = 800				
1	0.2	0.2	0.0	0.20000	0.19952	0.0011172	0.0011060	0.0011014	-0.00048	-0.0000046
2	0.2	0.2	0.2	0.16330	0.16283	0.0011268	0.0011369	0.0011320	-0.00047	-0.0000049
3	0.2	0.2	0.4	0.13093	0.13048	0.0011280	0.0011592	0.0011541	-0.00045	-0.0000052
4	0.2	0.4	0.0	0.20000	0.19949	0.0009709	0.0009680	0.0009644	-0.00051	-0.0000036
5	0.2	0.4	0.2	0.12247	0.12202	0.0010063	0.0010188	0.0010147	-0.00045	-0.0000041
6	0.2	0.4	0.4	0.04364	0.04327	0.0010181	0.0010460	0.0010416	-0.00038	-0.0000044
7	0.2	0.6	0.0	0.20000	0.19953	0.0007383	0.0007380	0.0007358	-0.00047	-0.0000022
8	0.2	0.6	0.2	0.08165	0.08126	0.0007808	0.0007894	0.0007868	-0.00039	-0.0000026
9	0.2	0.6	0.4	-0.04364	-0.04391	0.0007798	0.0007970	0.0007943	-0.00027	-0.0000027
10	0.4	0.2	0.2	0.36742	0.36679	0.0008814	0.0008988	0.0008966	-0.00064	-0.0000022
11	0.4	0.2	0.4	0.34915	0.34853	0.0008935	0.0009260	0.0009235	-0.00062	-0.0000025
12	0.4	0.2	0.6	0.35000	0.34940	0.0008843	0.0009248	0.0009223	-0.00060	-0.0000025
13	0.4	0.4	0.2	0.32660	0.32598	0.0008184	0.0008402	0.0008380	-0.00062	-0.0000022
14	0.4	0.4	0.4	0.26186	0.26131	0.0008756	0.0009119	0.0009090	-0.00055	-0.0000029
15	0.4	0.4	0.6	0.20000	0.19953	0.0009217	0.0009680	0.0009646	-0.00047	-0.0000034
16	0.4	0.6	0.2	0.28577	0.28524	0.0006629	0.0006777	0.0006760	-0.00053	-0.0000017
17	0.4	0.6	0.4	0.17457	0.17413	0.0007263	0.0007524	0.0007502	-0.00044	-0.0000022
18	0.4	0.6	0.6	0.05000	0.04969	0.0007620	0.0007960	0.0007935	-0.00031	-0.0000025
19	0.6	0.2	0.0	0.60000	0.59926	0.0004907	0.0004980	0.0004991	-0.00074	0.0000011
20	0.6	0.2	0.2	0.57155	0.57082	0.0005326	0.0005494	0.0005501	-0.00073	0.0000007
21	0.6	0.2	0.4	0.56737	0.56666	0.0005349	0.0005570	0.0005576	-0.00071	0.0000006

Table 6.2. Continued

	Conditions			r_{sp}			$var(r_{sp})$		Bias	
	ρ_{Y1}	ρ_{Y2}	ρ_{12}	Population Value	Empirical Mean	Empirical Variance	Population Value	Empirical Mean	r_{sp}	$var(r_{sp})$
22	0.6	0.4	0.2	0.53072	0.53002	0.0005356	0.0005577	0.0005575	-0.00070	-0.0000001
23	0.6	0.4	0.4	0.48008	0.47943	0.0006007	0.0006324	0.0006318	-0.00065	-0.0000006
24	0.6	0.4	0.6	0.45000	0.44942	0.0006436	0.0006760	0.0006750	-0.00058	-0.0000010
25	0.6	0.6	0.2	0.48990	0.48929	0.0004749	0.0004880	0.0004873	-0.00061	-0.0000007
26	0.6	0.6	0.4	0.39279	0.39225	0.0005574	0.0005829	0.0005818	-0.00055	-0.0000011
27	0.6	0.6	0.6	0.30000	0.29956	0.0006322	0.0006661	0.0006645	-0.00044	-0.0000016

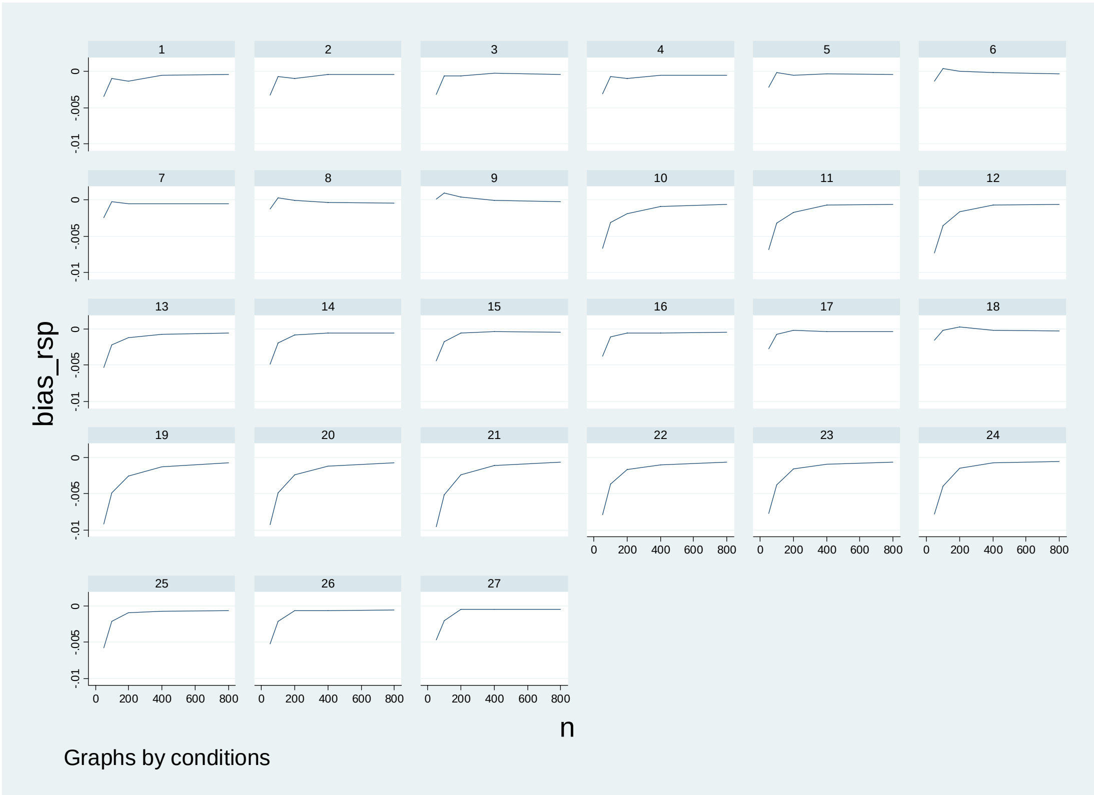

Figure 6.4. Bias of r_{sp} for Two Independent Variables

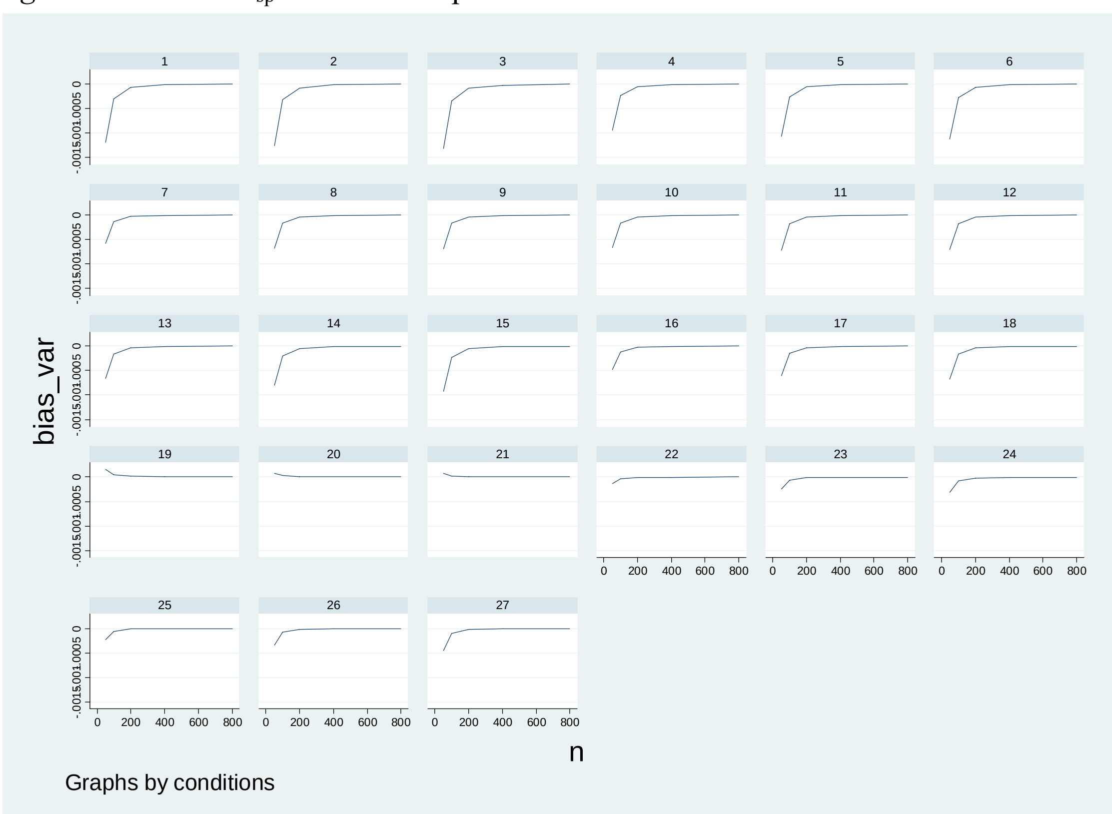

Figure 6.5. Bias of var(r_{sp}) for Two Independent Variables

Table 6.3. Analysis of Variance for Differences between r_{sp} and ρ_{sp} for $n = 50$, Two Predictors

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	2.853	1	2.853	211.569	$p < .001$
ρ_{Y1}	.611	2	.306	22.661	$p < .001$
ρ_{Y2}	.243	2	.121	8.994	$p < .001$
ρ_{12}	.020	3	.007	.493	$p = .687$
Error	1820.519	134992	.013		

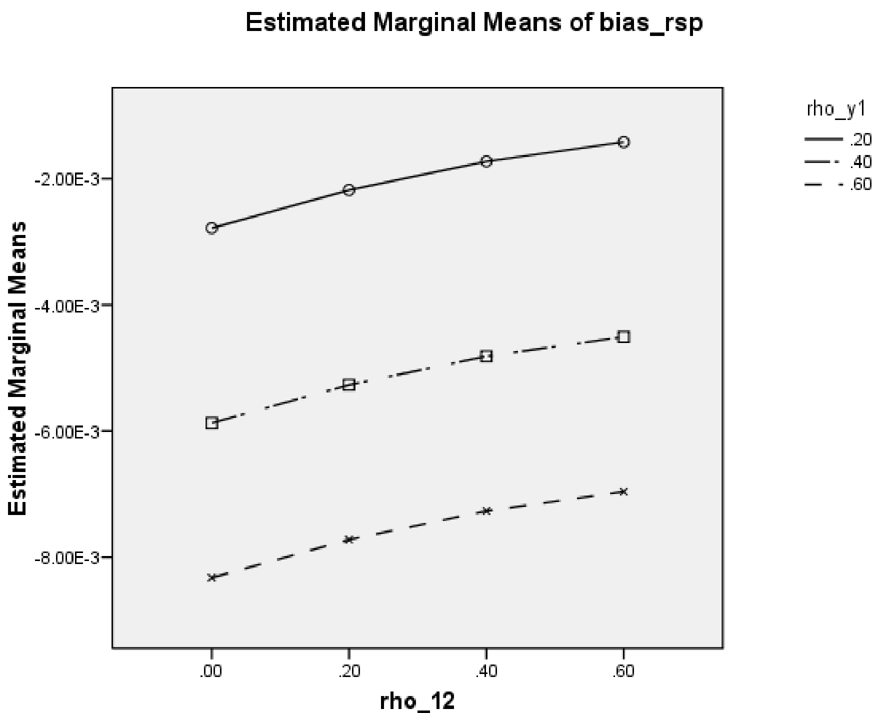

Figure 6.6. Average Bias of r_{sp} for Two Independent Variables ($n = 50$, by ρ_{Y1})

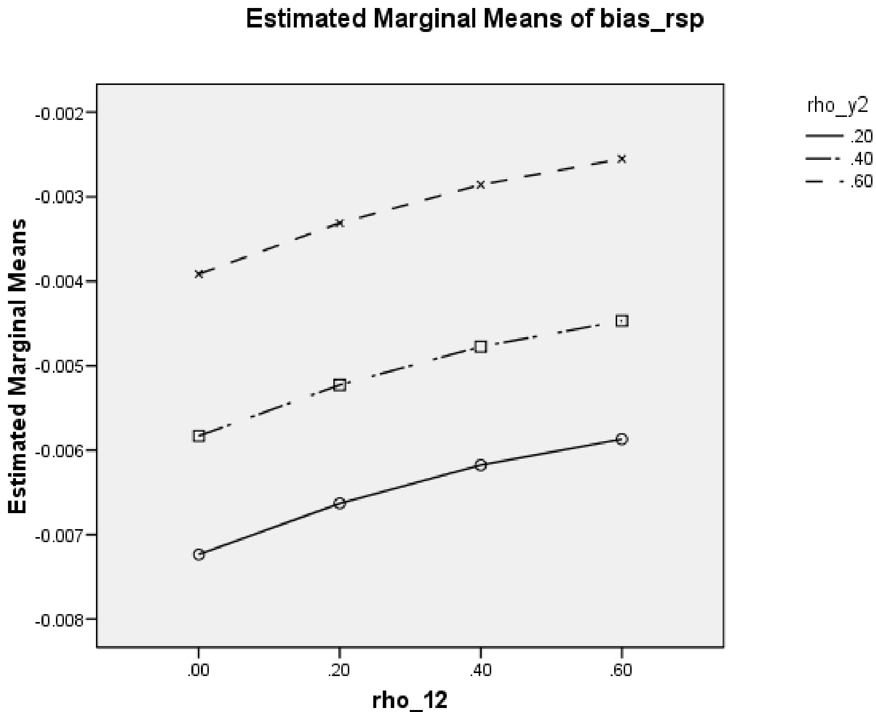

Figure 6.7. Average Bias of r_{sp} for Two Independent Variables (n = 50, by ρ_{Y2})

Table 6.4. Analysis of Variance for Differences between var(r_{sp}) and var(ρ_{sp}) for n = 50, Two Predictors

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	.039	1	.039	8860.468	$p < .001$
ρ_{Y1}	.015	2	.008	1719.298	$p < .001$
ρ_{Y2}	.000	2	.000	51.556	$p < .001$
ρ_{12}	.000	3	.000	34.884	$p < .001$
ρ_{Y1}*ρ_{Y2}	.003	4	.001	172.007	$p < .001$
ρ_{Y2}* ρ_{12}	8.26E-005	6	1.38E-005	3.149	$p = .004$
Error	.590	134982	4.37E-006		

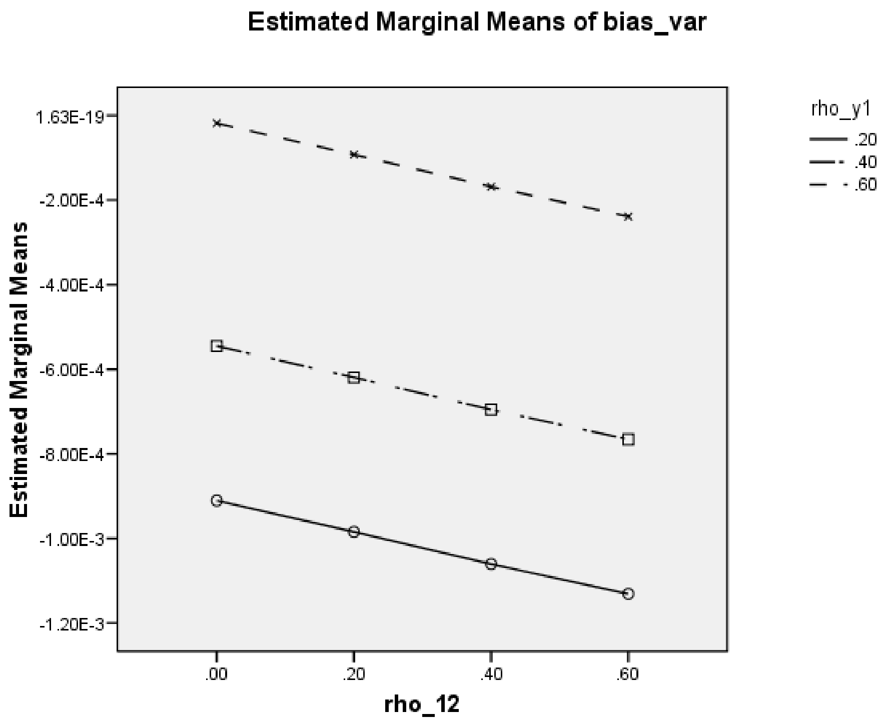

Figure 6.8. Average Bias of var(r_{sp}) for Two Independent Variables (n = 50, by ρ_{Y1})

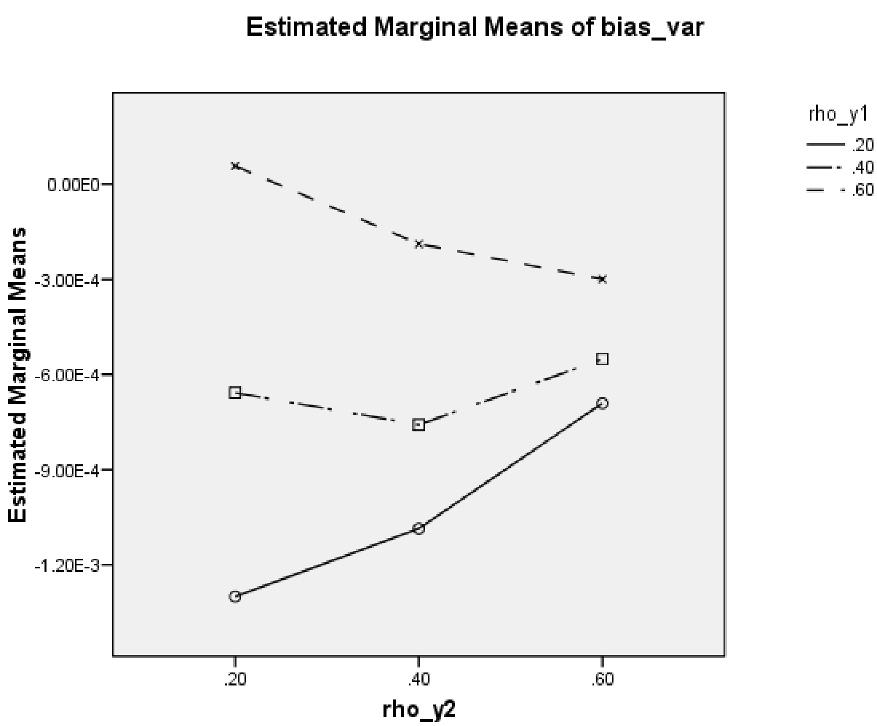

Figure 6.9. Average of Bias var(r_{sp}) for Two Independent Variables (n = 50, ρ_{Y1} and ρ_{Y2})

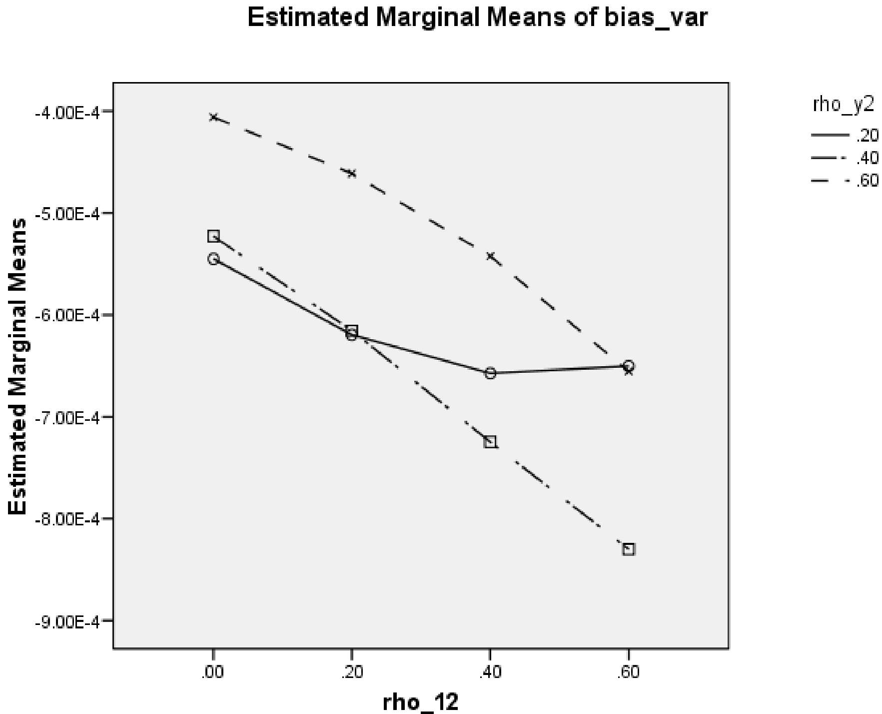

Figure 6.10. Average Bias of var(r_{sp}) for Two Independent Variables (n = 50, by ρ_{Y2})

The case with three independent variables

As was true for the case with two independent variables, from Table 6.5 we see that the estimators of r_{sp} and var(r_{sp}) underestimate the population values in the three predictor case. The biases of r_{sp} and var(r_{sp}) decrease when the sample size increases (see Figures 6.11 and 6.12). For instance, for condition 2 ($\rho_{Y1} = \rho_{Y2} = \rho_{Y3} = \rho_{13} = \rho_{23} = .2$, and $\rho_{12} = 0$) when the sample size is 50, the biases for r_{sp} and var(r_{sp}) are -0.006050 and -0.0026856, respectively. Then for the largest sample size (n = 800) the biases for r_{sp} and var(r_{sp}) reduce to -0.000820 and -0.0000511, respectively. Also, the same pattern is observed for the other conditions; this clearly indicates that when the sample size increases the biases for both r_{sp} and var(r_{sp}) decrease. For the larger sample size the biases reduce to virtually zero.

Next, I discuss the ANOVA analyses for the model with three independent variables for r_{sp} and var(r_{sp}). The factors for the model with three independent variables are ρ_{Y1}, ρ_{12}, ρ_{13}, and ρ_{23}, as was true for the model with two predictors, different analyses were run for each sample size to reduce heteroskedasticity. Nevertheless, for all the models Levene's test of homogeneity of variance was statistically significant at .001.

For $n = 50$, the results for r_{sp} indicate that only one of the four factors is statistically significant (ρ_{Y1}). No interaction effect is found among the factors. The adjusted R^2 for this model is also virtually zero, which indicates that although the F tests are statistical significant there is not really much explanatory power for the factors or interactions.

The main effect for ρ_{Y1} yields an F ratio of $F(1, 59993) = 30.02$, $p < .001$, indicating that the differences between r_{sp} values and their population values are affected by the correlation between the focal predictor and the outcome. None of the other factors are statistically significant (see Table 6.6). Figure 6.13 shows that for the larger value of ρ_{Y1} the bias is larger than for the smaller value.

For $n = 50$, the results for var(r_{sp}) indicate that the four main effects are statistically significant (ρ_{Y1}, ρ_{12}, ρ_{13}, and ρ_{23}). One interaction effect is found among the factors (ρ_{Y1} with ρ_{12}), as seen in Table 6.7. The adjusted R^2 for this model is .165.

The main effect for ρ_{Y1} yields an F ratio of $F(1, 59990) = 10013.31$, $p < .001$, indicating that the differences between var(r_{sp}) values and their population values are affected by the correlation among the focal predictor and the outcome. The main effect of ρ_{12} yields an F ratio of $F(3, 59990) = 98.19$, $p < .001$, indicating that the values are affected by the correlation between the predictor X_1 and X_2. The main effect of ρ_{13} yields an F ratio of $F(1, 59990) = 13.69$, $p < .001$, indicating that the values are also affected by the correlation between X_1 and X_3. The main effect of ρ_{23} yields an F ratio of $F(1, 59990) = 5.85$, $p = .016$, indicating that the values are also affected by the correlation between X_2 and X_3 (Figure 6.14). In addition, the interaction effect between ρ_{Y1} and ρ_{12} yields an F ratio of $F(3, 59990) = 5.70$, $p < .001$, indicating that the largest difference occurs when $\rho_{12} = .0$ (see Figure 6.15).

The ANOVAs for $n = 100$ and $n = 200$ for r_{sp} are virtually the same as for the sample size $n = 50$ (see Table A.9 and Table A.11, and Figure A.17 and Figure A.20, in Appendix A). For var(r_{sp}), results for $n = 200$ are virtually the same as for the sample size $n = 50$ (Table A.12. However, for $n = 100$ the interaction effect between ρ_{Y1} with ρ_{23}

is also significant is ρ_{Y1} with ρ_{12} with an F ratio of $F(1, 59990) = 17.627$, $p < .001$ (Table A.10 and Figures A.18 and A.19).

For $n = 400$, the results for differences between r_{sp} values and their population values indicated that none of the four factors (ρ_{Y1}, ρ_{12}, ρ_{13}, and ρ_{23}) are statistically significant (Table A.13 and Figure A.22, in Appendix A). No interaction effect is found among the factors and the adjusted R^2 for this model is again virtually zero. In addition, for var(r_{sp}) the only interaction effect that is significant is ρ_{Y1} with ρ_{12} (Table A.14 and Figure A.23, in Appendix A). In addition, for this model the variance explained is 53%, indicating that for this model the main factors and the interaction effect help to explain the variation into differences between var(r_{sp}) and its population values. The results for $n = 800$ are virtually the same as the results for $n = 400$ (Tables A.15 and A.16, and Figures A.24, A.25, and A.26, in Appendix A). The significant R^2 in the analyses of bias in var(r_{sp}) for the sample sizes 400 and 800 indicates that for larger sample sizes the significant main effects and interactions are large enough to have practical importance.

Table 6.5. Biases for r_{sp} and var(r_{sp}) for Three Independent Variables

Conditions					r_{sp}			var(r_{sp})		Bias	
	ρ_{Y1}	ρ_{12}	ρ_{13}	ρ_{23}	Population Value	Empirical Mean	Empirical Variance	Population Value	Empirical Mean	r_{sp}	var(r_{sp})
							n = 50				
1	0.2	0.0	0.1	0.2	0.18428	0.17675	0.0312412	0.0174218	0.0146738	-0.00753	-0.0027481
2	0.2	0.0	0.2	0.2	0.17079	0.16474	0.0271394	0.0175947	0.0149091	-0.00605	-0.0026856
3	0.2	0.1	0.1	0.1	0.16522	0.15990	0.0255673	0.0175479	0.0147561	-0.00532	-0.0027918
4	0.2	0.2	0.2	0.2	0.13784	0.13386	0.0179189	0.0179581	0.0150444	-0.00398	-0.0029138
5	0.2	0.4	0.1	0.1	0.11929	0.11599	0.0134544	0.0180216	0.0149307	-0.00330	-0.0030910
6	0.2	0.4	0.2	0.2	0.11009	0.10713	0.0114760	0.0182171	0.0151384	-0.00296	-0.0030787
7	0.4	0.0	0.1	0.2	0.38534	0.37236	0.1386505	0.0135640	0.0131158	-0.01299	-0.0004481
8	0.4	0.0	0.2	0.2	0.37417	0.36357	0.1321868	0.0138272	0.0133650	-0.01059	-0.0004622
9	0.4	0.1	0.1	0.1	0.36699	0.35620	0.1268761	0.0139128	0.0132866	-0.01079	-0.0006262
10	0.4	0.2	0.2	0.2	0.34514	0.33511	0.1122974	0.0145032	0.0136568	-0.01003	-0.0008465
11	0.4	0.4	0.1	0.1	0.33798	0.32834	0.1078069	0.0145695	0.0135606	-0.00964	-0.0010089
12	0.4	0.4	0.2	0.2	0.33027	0.32093	0.1029986	0.0148323	0.0137917	-0.00934	-0.0010406
							n = 100				
1	0.2	0.0	0.1	0.2	0.18428	0.18102	0.0327676	0.0087109	0.0078222	-0.00326	-0.0008887
2	0.2	0.0	0.2	0.2	0.17079	0.16885	0.0285113	0.0087973	0.0079529	-0.00194	-0.0008445
3	0.2	0.1	0.1	0.1	0.16522	0.16357	0.0267537	0.0087739	0.0078698	-0.00165	-0.0009041
4	0.2	0.2	0.2	0.2	0.13784	0.13684	0.0187244	0.0089791	0.0080315	-0.00100	-0.0009476
5	0.2	0.4	0.1	0.1	0.11929	0.11798	0.0139198	0.0090108	0.0079711	-0.00131	-0.0010398
6	0.2	0.4	0.2	0.2	0.11009	0.10904	0.0118897	0.0091085	0.0080863	-0.00105	-0.0010222
7	0.4	0.0	0.1	0.2	0.38534	0.37945	0.1439859	0.0067820	0.0069440	-0.00589	0.0001620
8	0.4	0.0	0.2	0.2	0.37417	0.37049	0.1372616	0.0069136	0.0070826	-0.00368	0.0001690
9	0.4	0.1	0.1	0.1	0.36699	0.36278	0.1316105	0.0069564	0.0070421	-0.00421	0.0000857
10	0.4	0.2	0.2	0.2	0.34514	0.34110	0.1163458	0.0072516	0.0072508	-0.00405	-0.0000008
11	0.4	0.4	0.1	0.1	0.33798	0.33371	0.1113647	0.0072847	0.0072025	-0.00427	-0.0000822
12	0.4	0.4	0.2	0.2	0.33027	0.32623	0.1064249	0.0074161	0.0073307	-0.00404	-0.0000854

Table 6.5. Continued

	Conditions				r_{sp}			$var(r_{sp})$		Bias	
					Population Value	Empirical Mean	Empirical Variance	Population Value	Empirical Mean		
	ρ_{Y1}	ρ_{12}	ρ_{13}	ρ_{23}						r_{sp}	$var(r_{sp})$
							$n = 200$				
1	0.2	0.0	0.1	0.2	0.18428	0.18252	0.0333154	0.0043555	0.0040342	-0.00176	-0.0003212
2	0.2	0.0	0.2	0.2	0.17079	0.17025	0.0289847	0.0043987	0.0041033	-0.00054	-0.0002953
3	0.2	0.1	0.1	0.1	0.16522	0.16514	0.0272726	0.0043870	0.0040587	-0.00008	-0.0003283
4	0.2	0.2	0.2	0.2	0.13784	0.13840	0.0191549	0.0044895	0.0041434	0.00056	-0.0003461
5	0.2	0.4	0.1	0.1	0.11929	0.11983	0.0143587	0.0045054	0.0041103	0.00054	-0.0003951
6	0.2	0.4	0.2	0.2	0.11009	0.11080	0.0122762	0.0045543	0.0041711	0.00071	-0.0003832
7	0.4	0.0	0.1	0.2	0.38534	0.38218	0.1460619	0.0033910	0.0035709	-0.00316	0.0001799
8	0.4	0.0	0.2	0.2	0.37417	0.37314	0.1392339	0.0034568	0.0036442	-0.00103	0.0001874
9	0.4	0.1	0.1	0.1	0.36699	0.36560	0.1336655	0.0034782	0.0036215	-0.00138	0.0001433
10	0.4	0.2	0.2	0.2	0.34514	0.34397	0.1183125	0.0036258	0.0037305	-0.00117	0.0001047
11	0.4	0.4	0.1	0.1	0.33798	0.33697	0.1135504	0.0036424	0.0037028	-0.00101	0.0000604
12	0.4	0.4	0.2	0.2	0.33027	0.32942	0.1085145	0.0037081	0.0037703	-0.00086	0.0000622
							$n = 400$				
1	0.2	0.0	0.1	0.2	0.18428	0.18225	0.0332133	0.0021777	0.0020495	-0.00204	-0.0001282
2	0.2	0.0	0.2	0.2	0.17079	0.16991	0.0288679	0.0021993	0.0020851	-0.00089	-0.0001142
3	0.2	0.1	0.1	0.1	0.16522	0.16478	0.0271529	0.0021935	0.0020622	-0.00044	-0.0001313
4	0.2	0.2	0.2	0.2	0.13784	0.13791	0.0190181	0.0022448	0.0021060	0.00007	-0.0001388
5	0.2	0.4	0.1	0.1	0.11929	0.11929	0.0142297	0.0022527	0.0020891	0.00000	-0.0001636
6	0.2	0.4	0.2	0.2	0.11009	0.11020	0.0121448	0.0022771	0.0021204	0.00011	-0.0001567
7	0.4	0.0	0.1	0.2	0.38534	0.38267	0.1464372	0.0016955	0.0018119	-0.00267	0.0001164
8	0.4	0.0	0.2	0.2	0.37417	0.37359	0.1395724	0.0017284	0.0018497	-0.00057	0.0001213
9	0.4	0.1	0.1	0.1	0.36699	0.36601	0.1339635	0.0017391	0.0018381	-0.00098	0.0000990
10	0.4	0.2	0.2	0.2	0.34514	0.34426	0.1185174	0.0018129	0.0018944	-0.00088	0.0000815
11	0.4	0.4	0.1	0.1	0.33798	0.33723	0.1137259	0.0018212	0.0018804	-0.00075	0.0000592
12	0.4	0.4	0.2	0.2	0.33027	0.32964	0.1086592	0.0018540	0.0019152	-0.00064	0.0000612

Table 6.5. Continued

Conditions					r_{sp}			var(r_{sp})		Bias	
	ρ_{Y1}	ρ_{12}	ρ_{13}	ρ_{23}	Population Value	Empirical Mean	Empirical Variance	Population Value	Empirical Mean	r_{sp}	var(r_{sp})
							n = 800				
1	0.2	0.0	0.1	0.2	0.18428	0.18255	0.0333260	0.0010889	0.0010303	-0.00173	-0.0000585
2	0.2	0.0	0.2	0.2	0.17079	0.16997	0.0288902	0.0010997	0.0010485	-0.00082	-0.0000511
3	0.2	0.1	0.1	0.1	0.16522	0.16496	0.0272119	0.0010967	0.0010368	-0.00026	-0.0000600
4	0.2	0.2	0.2	0.2	0.13784	0.13775	0.0189757	0.0011224	0.0010591	-0.00009	-0.0000633
5	0.2	0.4	0.1	0.1	0.11929	0.11909	0.0141824	0.0011264	0.0010505	-0.00020	-0.0000759
6	0.2	0.4	0.2	0.2	0.11009	0.10984	0.0120643	0.0011386	0.0010665	-0.00025	-0.0000721
7	0.4	0.0	0.1	0.2	0.38534	0.38310	0.1467679	0.0008477	0.0009104	-0.00224	0.0000627
8	0.4	0.0	0.2	0.2	0.37417	0.37380	0.1397229	0.0008642	0.0009298	-0.00037	0.0000656
9	0.4	0.1	0.1	0.1	0.36699	0.36633	0.1341954	0.0008695	0.0009237	-0.00066	0.0000542
10	0.4	0.2	0.2	0.2	0.34514	0.34428	0.1185267	0.0009065	0.0009524	-0.00086	0.0000460
11	0.4	0.4	0.1	0.1	0.33798	0.33727	0.1137510	0.0009106	0.0009452	-0.00071	0.0000346
12	0.4	0.4	0.2	0.2	0.33027	0.32951	0.1085776	0.0009270	0.0009630	-0.00076	0.0000360

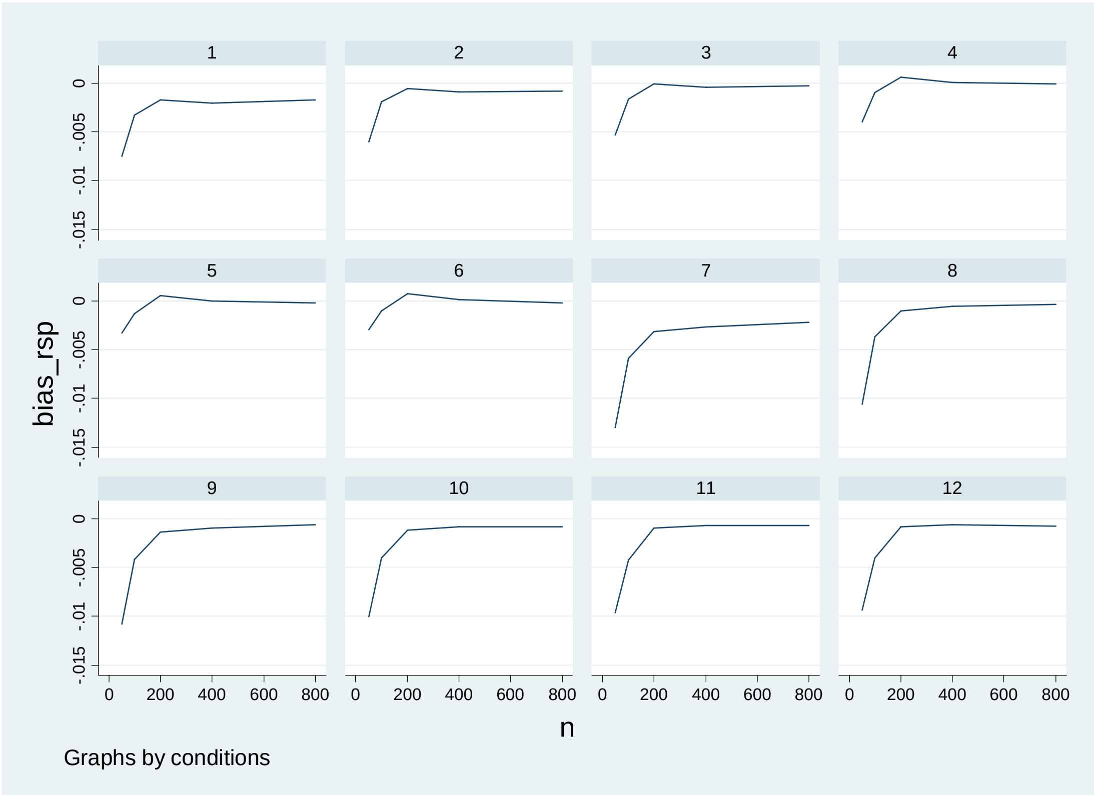

Figure 6.11. Bias of r_{sp} for Three Independent Variables

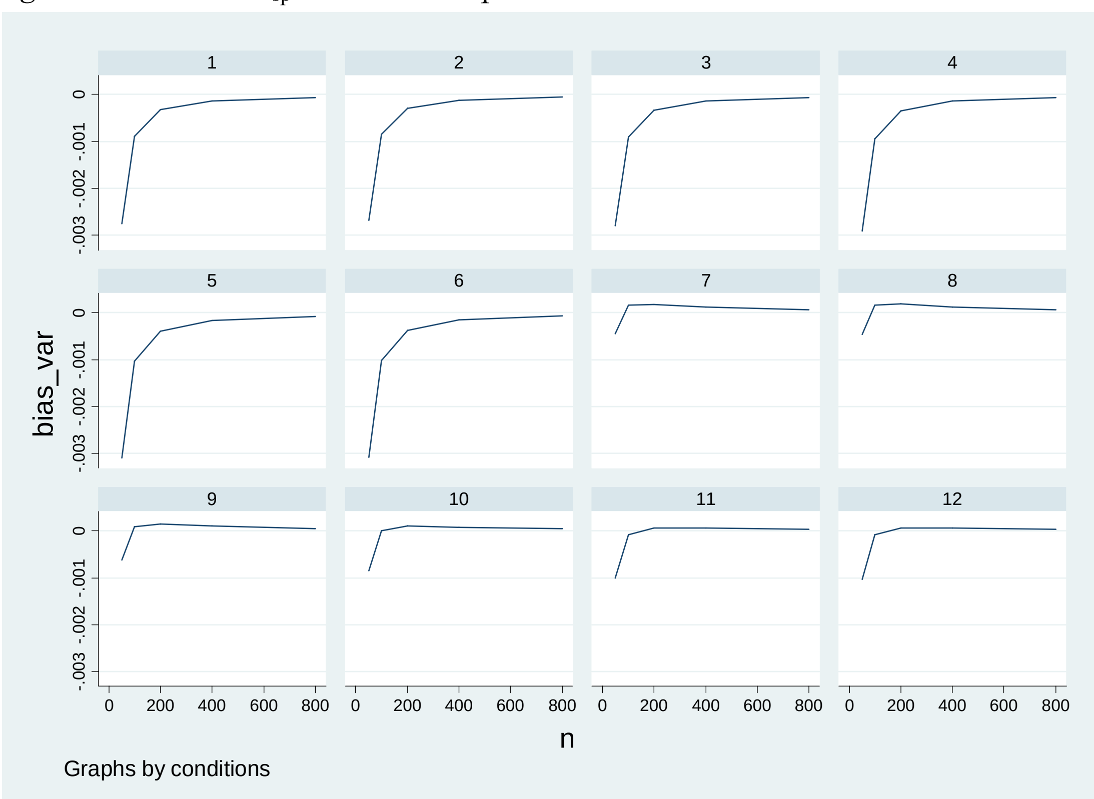

Figure 6.12. Bias of var(r_{sp}) for Three Independent Variables

Table 6.6. Analysis of Variance for Differences between r_{sp} and ρ_{sp} for n = 50, Three Predictors

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	2.228	1	2.228	137.081	$p < .001$
ρ_{Y1}	.488	1	.488	30.025	$p < .001$
ρ_{12}	.036	3	.012	.749	$p = .523$
ρ_{13}	.001	1	.001	.040	$p = .841$
ρ_{23}	5.00E-006	1	5.00E-006	.000	$p = .986$
Error	974.962	59993	.016		

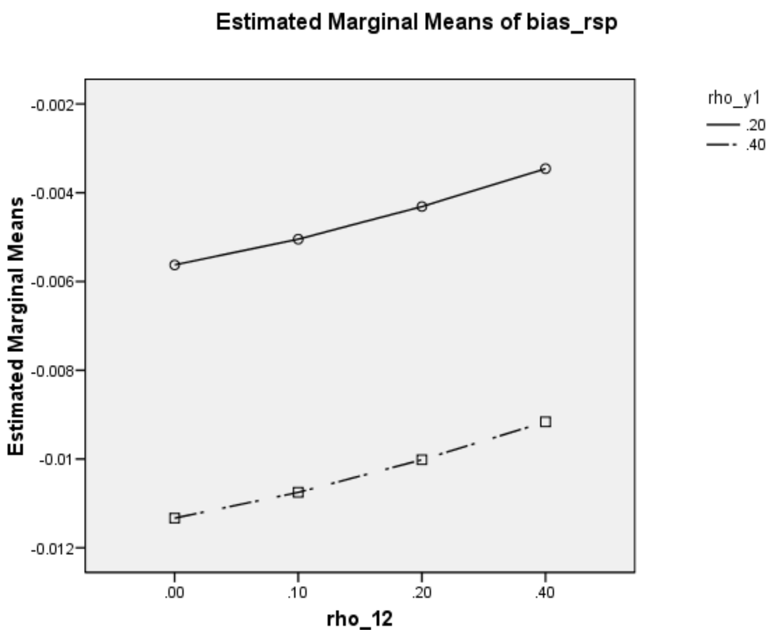

Figure 6.13. Average Bias of r_{sp} for Three Independent Variables (n = 50, by ρ_{Y1})

Table 6.7. Analysis of Variance for Differences between var(r_{sp}) and var(ρ_{sp}) for n = 50, Three Predictors

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	.130	1	.130	21429.325	$p < .001$
ρ_{Y1}	.061	1	.061	10013.310	$p < .001$
ρ_{12}	.002	3	.001	98.195	$p < .001$
ρ_{13}	8.31E-005	1	8.31E-005	13.697	$p < .001$
ρ_{23}	3.56E-005	1	3.56E-005	5.858	$p = .016$
$\rho_{Y1} * \rho_{Y2}$	.0001	3	3.47E-005	5.709	$p < .001$
Error	.364	59990	6.07E-006		

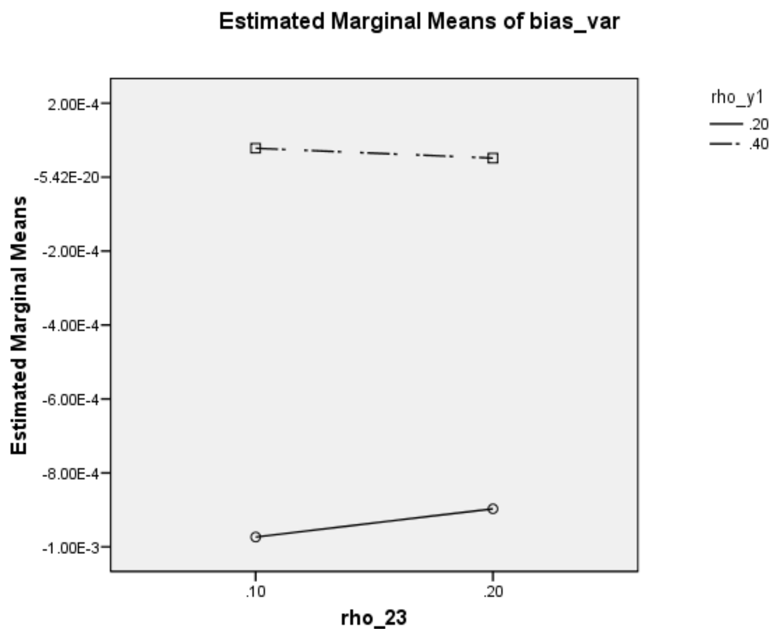

Figure 6.14. Average Bias of var(r_{sp}) for Three Independent Variables (n = 50, by ρ_{Y1})

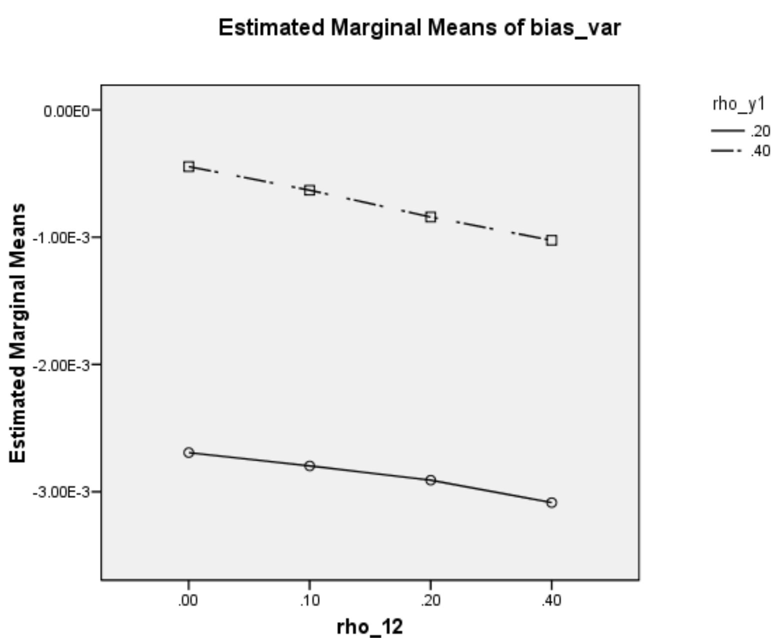

Figure 6.15. Average Bias of var(r_{sp}) for Three Independent Variables (n = 50, by ρ_{Y1})

The cases with five and ten independent variables

Only one condition was run for the models with five and ten independent variables models. The goal of running these two models was to examine the effect of the number of predictors on the estimators of r_{sp} and var(r_{sp}). Thus, all the correlations among the predictors and the correlations between the predictors and the dependent variable were set to .2. Table 6.8 presents the biases for r_{sp} and var(r_{sp}) for the models for five and ten independent variables. As found for the cases with two and three predictors, the biases of r_{sp} and var(r_{sp}) decrease when the sample sizes increase (see Figures 6.16 and 6.17). For instance, for the model with five predictors when the sample size is 50, the biases for r_{sp} and var(r_{sp}) are -0.005818 and -0.0045851, respectively. Then for the largest sample size (n = 800) for the model with five predictors the biases for r_{sp} and var(r_{sp}) reduce to -0.000034 and -0.0001124, respectively. The same pattern is observed for the model with ten independent variables.

Next, I discuss the ANOVA analyses for the models with all intercorrelations equal .2, for two, three, five, and ten independent variables. The factors are the sample size (n) and number of predictors in the model (p). The analyses are performed to study the impact of the number of predictors into the bias of r_{sp} and var(r_{sp}). Thus, the conditions for two and three predictors in which all the correlations among the predictors and the correlations between the predictors and the dependent variable were set to .2 are included in the analyses.

The results for the differences between r_{sp} and ρ_{sp} indicate that the two factors (sample size and number of predictors) are statistically significant. No interaction effect is found between the factors. The adjusted R^2 for this model is also virtually zero, which indicates that although the F tests are statistically significant there is not really much practical difference among the bias values across the factor levels.

The main effect for sample size (n) yields an F ratio of $F(4, 99980) = 13.47$, $p < .001$, indicating that the differences for r_{sp} values are affected by the sample size (Table 6.9). The main effect for number of predictors yields an F ratio of $F(3, 99980) = 33.62$, $p < .001$, indicating that the differences between r_{sp} values and its population values are affected by the number of predictors in the model. Figure 6.18 shows that the largest bias occurs for the case with 10 predictors and the smallest sample size.

For the differences between var(r_{sp}) values and its population value, the results indicate that the two main effects and its interaction are statistically significant (sample size and case). The adjusted R^2 for this model is .798. This is the largest R^2 value across all analyses, indicating that for the bias of var(r_{sp}) the number of predictors and the sample size are determining important factors.

The main effect for sample size yields an F ratio of $F(4, 99980) = 39298.68$, $p < .001$, indicating that the differences involving var(r_{sp}) values are affected by the sample size (Table 6.10). The main effect of number of predictors yields an F ratio of $F(3, 99980) = 12314.91$, $p < .001$, indicating that the differences between var(r_{sp}) and their population values are affected by the number of predictors in the model. In addition, the interaction effect between sample size and number of predictors yields an F ratio of $F(12, 99980) = 4045.58$, $p < .001$, indicating that when the number of predictors increases, the bias is larger for smaller sample sizes, though this difference virtually disappears for larger sample sizes (see Figure 6.19).

Table 6.8. Biases for r_{sp} and var(r_{sp}) for Five and Ten Independent Variables

	r_{sp}			var(r_{sp})		Bias	
Cases	Population Value	Empirical Mean	Empirical Variance	Population Value	Empirical Mean	r_{sp}	var(r_{sp})
			$n = 50$				
5 IV	0.10488	0.09906	0.017495	0.0176064	0.013021	-0.00582	-0.0045851
10 IV	0.07070	0.05756	0.017842	0.0170483	0.009653	-0.01314	-0.0073954
			$n = 100$				
5 IV	0.10488	0.10346	0.009031	0.0088032	0.007243	-0.00142	-0.0015599
10 IV	0.07070	0.06346	0.008668	0.0085241	0.00601	-0.00724	-0.0025142
			$n = 200$				
5 IV	0.10488	0.10408	0.004531	0.0044016	0.003802	-0.00080	-0.0005991
10 IV	0.07070	0.06370	0.004197	0.0042621	0.003344	-0.00700	-0.0009177
			$n = 400$				
5 IV	0.10488	0.10427	0.002174	0.0022008	0.001954	-0.00061	-0.0002472
10 IV	0.07070	0.06653	0.002124	0.0021310	0.001758	-0.00417	-0.0003729
			$n = 800$				
5 IV	0.10488	0.10485	0.001091	0.0011004	0.000988	-0.00003	-0.0001124
10 IV	0.07070	0.06565	0.001053	0.0010655	0.000902	-0.00505	-0.0001632

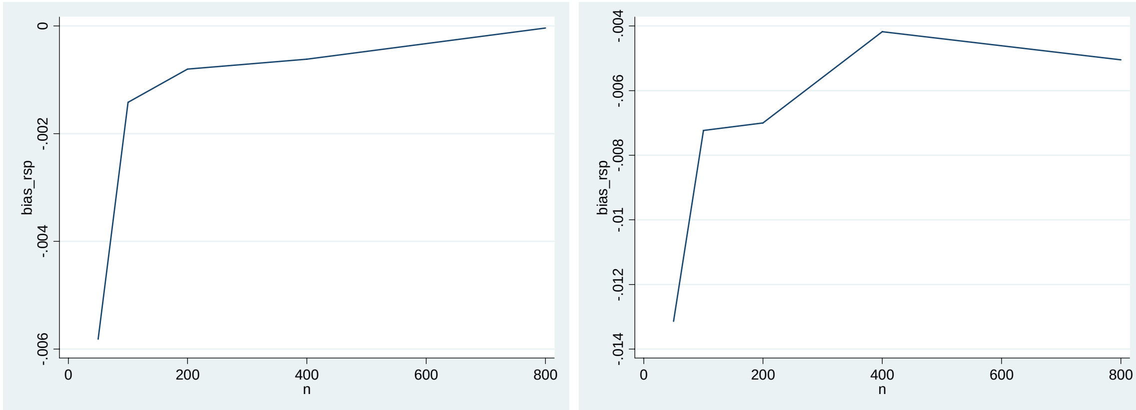

Figure 6.16. Bias of r_{sp} for Five and Ten Independent Variables

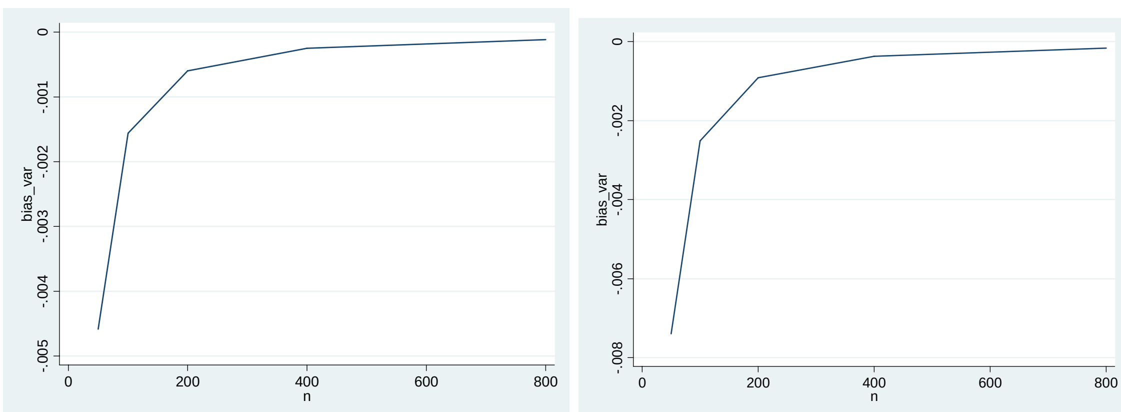

Figure 6.17. Bias of var(r_{sp}) for Five and Ten Independent Variables

Table 6.9. Analysis of Variance for Differences between r_{sp} and ρ_{sp} for Two, Three, Five, and Ten Predictors

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	.776	1	.776	111.340	$p < .001$
n	.376	4	.094	13.473	$p < .001$
p	.703	3	.234	33.625	$p < .001$
n * *p*	.070	12	.006	.837	$p = .613$
Error	696.884	99980	.007		

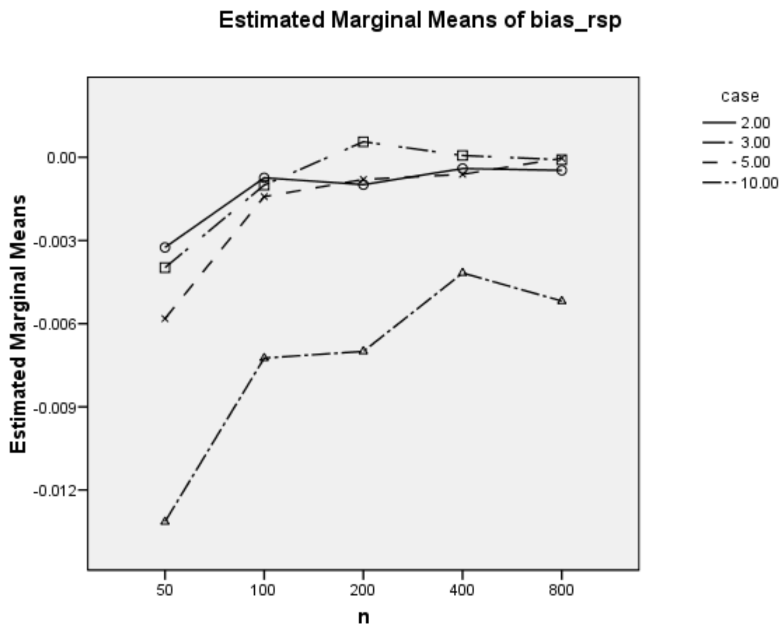

Figure 6.18. Average Bias of r_{sp} for Two, Three, Five, and Ten Predictors

Table 6.10. Analysis of Variance for Differences between var(r_{sp}) and var(ρ_{sp}) for Two, Three, Five, and Ten Predictors

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	.151	1	.151	109379.974	$p < .001$
n	.217	4	.054	39298.688	$p < .001$
p	.051	3	.017	12314.912	$p < .001$
n * *p*	.067	12	.006	4045.581	$p < .001$
Error	.138	99980	1.38E-006		

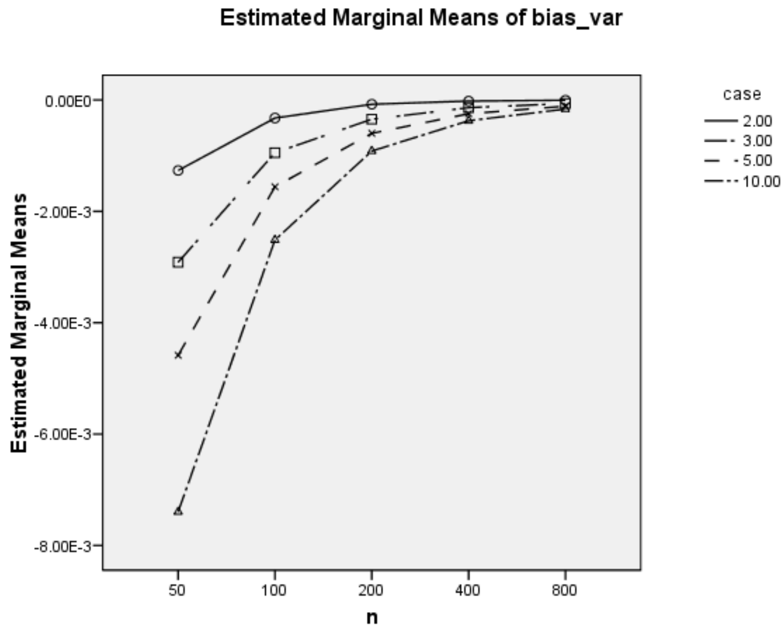

Figure 6.19. Average Bias of var(r_{sp}) for Two, Three, Five, and Ten Predictors

CHAPTER 7

DISCUSSION

In this chapter I present a summary of the simulation results. Then, I provide suggestions for the analysis of r_{sp} values. Limitations and future directions are also discussed.

When estimating effect sizes from multiple regression models it may be of interest to partial out a variable or set of variables from the key predictors present in the model (Aloe & Becker, in press). For instance, a researcher may be interested in the increment in the proportion of variance in students' academic achievement accounted for by their teacher's verbal ability.

This dissertation presents an index to estimate the semi-partial correlation, and a formula for its variance, for when the correlation matrix is not presented in a primary study. The r_{sp} index can be seen as a partial effect in the r family. In a multiple regression when the predictors are completely uncorrelated, r_{sp} will be identical to the bivariate correlation. On the other hand, when predictors are interrelated (as they typically are in social science), r_{sp} represents the unique contribution to the outcome Y made by the focal predictor (the predictor of interest). What is innovative about this index is that it will enable reviewers to now include effects representing more complex studies into meta-analyses.

In the current research, two estimators (r_{sp} and var(r_{sp})) are studied under different conditions – according to sample size, correlations among predictors and between the predictors and dependent variable, and number of predictors. Specifically, the results presented in Chapter 6 indicate that both r_{sp} and var(r_{sp}) are affected by the sample size, the correlations among the predictors, and the number of predictors in the model. The major finding of this study is that the r_{sp} index can be used to estimate the semi-partial correlation using statistics usually reported in primary studies, with very little bias.

In addition, the results of this study indicated that both estimators r_{sp} and var(r_{sp}) behave as expected, under smaller samples the biases of both r_{sp} and var(r_{sp}) are larger than under larger sample sizes. For several of the conditions examined the bias is virtually zero for both r_{sp} and var(r_{sp}). The analyses of variance indicated that for the differences between r_{sp} and ρ_{sp}, basically no variance was explained by the true correlations among predictors and between the predictors and the dependent variable. On the other hand, for var(r_{sp}), for the larger sample size at least 50% of the variation in differences between var(r_{sp}) and its population value is explained by the factors. This suggests that the bias in var(r_{sp}) is affected by the true correlations among predictors and between the predictors and the dependent variable.

With the variance derived in this dissertation, meta-analysis of the semi-partial correlations can proceed in a similar fashion to typical meta-analyses, such as the methods described by Hedges and Olkin (1985) or in the *The handbook of research synthesis* (Cooper & Hedges, 1994b). Standard errors and confidence intervals can be computed for the individual r_{sp} values, and weighted analyses can be used to explore heterogeneity and to estimate central tendency and variation in the effects. However, the behavior of meta-analytic summaries computed using r_{sp} has not been investigated in this dissertation.

Taking into consideration that each semi-partial correlation may arise from a different regression model, it is conceivable that each r_{sp} in a meta-analysis may be estimating a different population parameter. The inclusion of other predictors in the model and the number of predictors in the model may affect the true value of the r_{sp}. Therefore, random-effects modeling (Hedges & Vevea, 1998) may be more appropriate than a fixed-effects approach. Furthermore, when analyzing a set of r_{sp} values, the reviewer needs to include predictor variables that reflect the differences in those models. For instance, it is typical in educational research for primary studies to control for socio-economic status and race. Thus, it is recommended the meta-analyst code the number of predictors in each regression model, which may represent the degree to which the individual r_{sp} values differ from the zero-order correlation, and from each other. Then, reviewers should code indicator (or “dummy”) variables representing the presence or absence of key covariates or control variables in each study’s regression model. For instance, in a synthesis of the studies of teachers’ verbal ability and student achievement, an important control variable is the student’s prior achievement. If this is not controlled, the primary-study regression analyses cannot give a strong indication of the importance of the *current* teacher's verbal ability for student achievement. Without controlling for inequalities in prior achievement levels, variation in current achievement levels may reflect contributions to the student's current level of performance made by any or all prior teachers. When the coded variables are important predictors of the outcome of interest, r_{sp} values from models that include those variables will be lower than zero-order *r*s, and may also be lower than other semi-partial correlations (from models without those control variables). Consequently, the more valuable control variables that are included in the model, the lower the semi-partial *r* may be for the focal predictor (if the predictors included are intercorrelated).

A last consideration for the analysis of r_{sp} values is whether to combine them with zero-order correlations. Technically, the semi-partial correlation is estimating a different parameter than the bivariate *r*. In addition, the results of this dissertation confirm that when the correlations among the predictors increase the true value of the r_{sp} decreases. Moreover, when the model includes more predictors that are intercorrelated, the value of the true value of the r_{sp} decreases as well. For instance, when all the intercorrelations were .2 the true r_{sp} values decreases from .16 for the model with two predictors to .07 for the model with ten predictors. In addition, when the number of predictors is constant and the intercorrelations among predictors vary the true r_{sp} values decreases when the intercorrelation among the predictors increases. Specifically, for the model with two predictors the true r_{sp} values range from -0.04 to 0.6 depending the correlation between the predictors and the outcome and among predictors.

Thus, reviewers should use caution in combining effects computed using the r_{sp} formula with bivariate correlations, because r_{sp} values tend to be smaller than bivariate correlations. When the decision of combining *r* and r_{sp} indices in the analysis is made, variables representing the complexity of the models from which the r_{sp}s were drawn must be incorporated into analyses of the full set of effects. In such analyses, for instance, the variable "number of predictors" would be coded as 1 for outcomes that are bivariate *r*s, since the bivariate *r* represents a regression model with just one predictor. However, this dissertation does not examine the combination of *r* and r_{sp} indices. Thus, a next step will be to directly study the effects of combining r_{sp} indices in the weighted mean, standard error and so on. Although this dissertation is the first step towards meta-analysis of the r_{sp} index, further consideration should be given to the use of this index in the homogeneity test (*Q* statistic), and the effect of moderators in the analysis.

Finally, while one contribution of this dissertation is to present an index to estimate a partial effect size for the *r* family of effects from results that are typically reported in multiple regression analysis, the major contribution of this dissertation is the derivation of the variance for this index. The formula for var(r_{sp}) allows researchers to proceed using inverse variance weights in a similar fashion to typical meta-analyses. The major limitation of this dissertation is the number of conditions studied in the simulation. However, the conditions used in this dissertation cover a reasonable range of possible conditions and are consistent and clearly show the behavior of both estimators (r_{sp} and var(r_{sp})).

APPENDIX A

ANOVAS RESULTS

Table A.1. Analysis of Variance for Differences between r_{sp} and ρ_{sp} for n = 100, Two Predictors

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	.380(a)	7	.054	8.340	.000
Intercept	.463	1	.463	71.072	.000
ρ_{Y1}	.254	2	.127	19.484	.000
ρ_{Y2}	.101	2	.050	7.752	.000
ρ_{12}	.005	3	.002	.243	.867
Error	879.003	134992	.007		
Total	879.887	135000			
Corrected Total	879.383	134999			

a R Squared = .000 (Adjusted R Squared = .000)

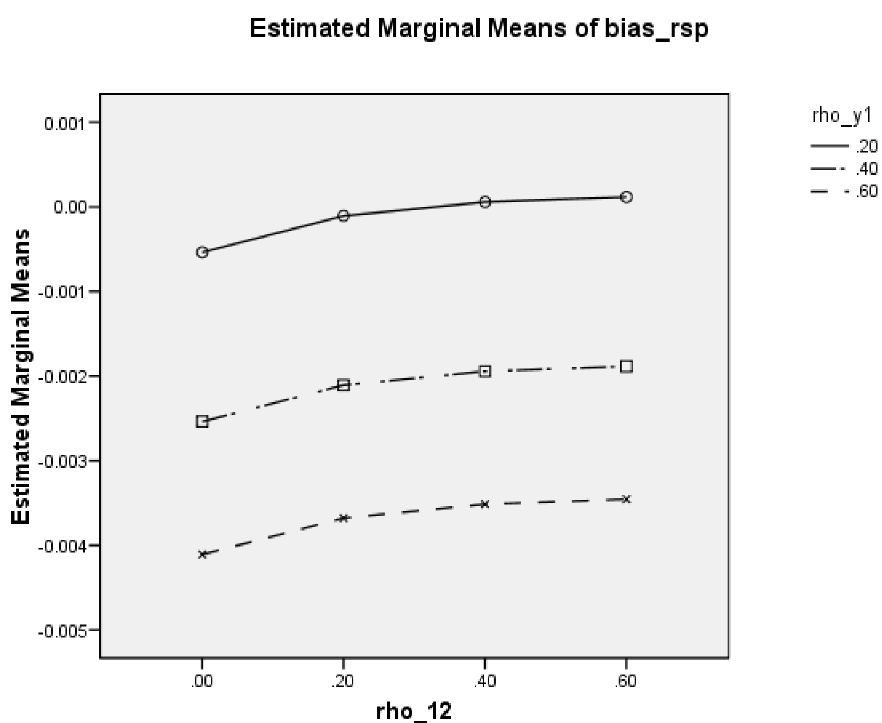

Figure A.1. Average Bias of r_{sp} for Two Independent Variables (n = 100, by ρ_{Y1})

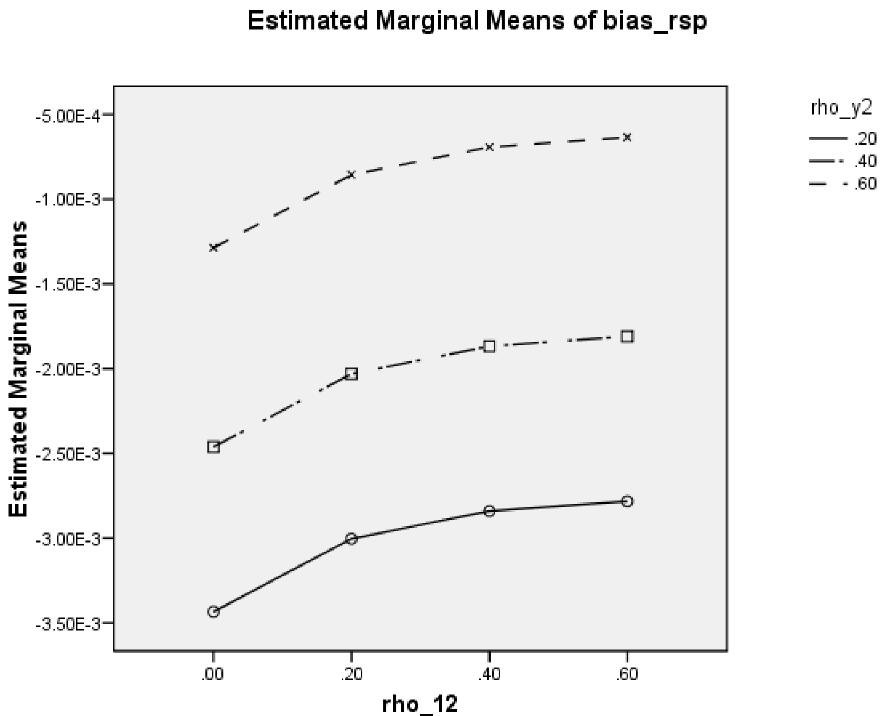

Figure A.2. Average Bias of r_{sp} for Two Independent Variables (n = 100, by ρ_{Y2})

Table A.2. Analysis of Variance for Differences between var(r_{sp}) and var(ρ_{sp}) for n = 100, Two Predictors

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	.001(a)	11	.000	226.662	.000
Intercept	.003	1	.003	4595.805	.000
ρ_{Y1}	.001	2	.001	916.072	.000
ρ_{Y2}	5.22E-005	2	2.61E-005	45.396	.000
ρ_{12}	3.67E-005	3	1.22E-005	21.302	.000
$\rho_{Y1}*\rho_{Y2}$	.000	4	6.32E-005	109.947	.000
Error	.078	134988	5.75E-007		
Total	.082	135000			
Corrected Total	.079	134999			

a R Squared = .018 (Adjusted R Squared = .018)

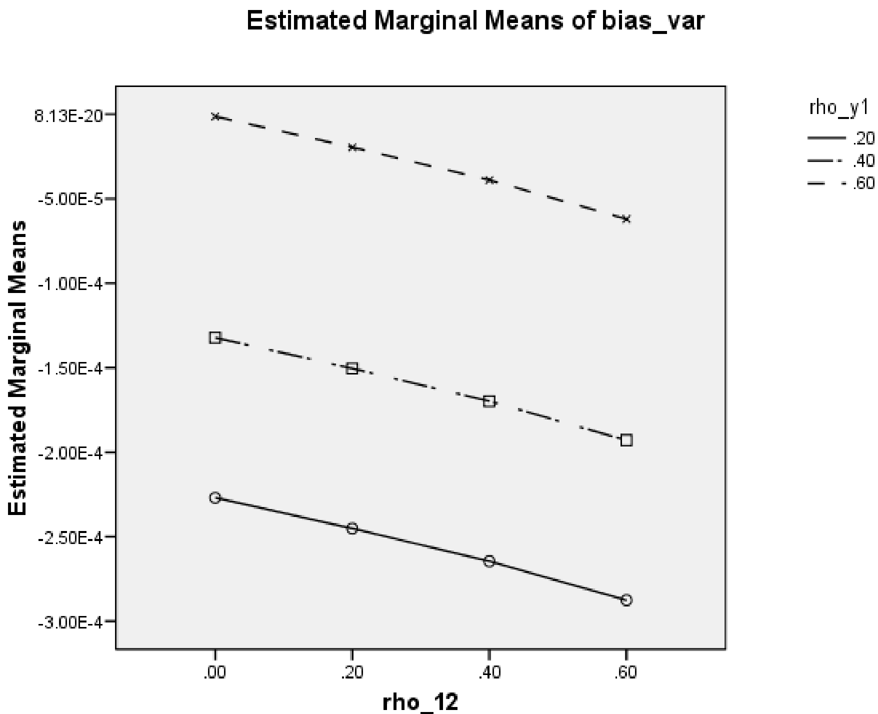

Figure A.3. Average Bias of var(r_{sp}) for Two Independent Variables (n = 100, by ρ_{Y1})

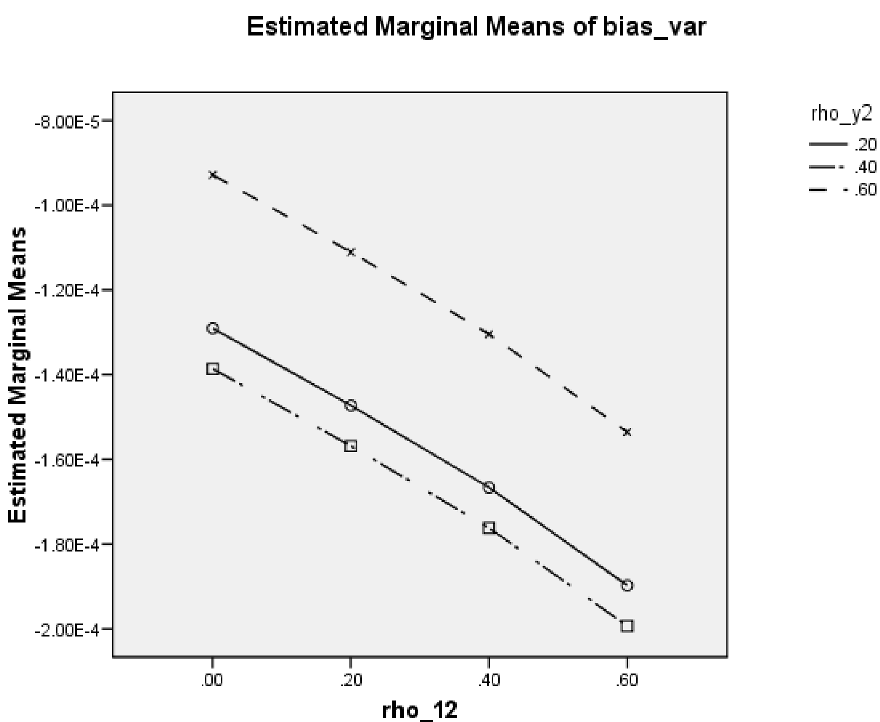

Figure A.4. Average Bias of var(r_{sp}) for Two Independent Variables (n = 100, by ρ_{Y1})

Table A.3. Analysis of Variance for Differences between r_{sp} and ρ_{sp} for n = 200, Two Predictors

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	.079(a)	7	.011	3.530	.001
Intercept	.128	1	.128	39.818	.000
ρ_{Y1}	.031	2	.016	4.861	.008
ρ_{Y2}	.038	2	.019	5.853	.003
ρ_{12}	.009	3	.003	.945	.418
Error	432.568	134992	.003		
Total	432.784	135000			
Corrected Total	432.648	134999			

a R Squared = .000 (Adjusted R Squared = .000)

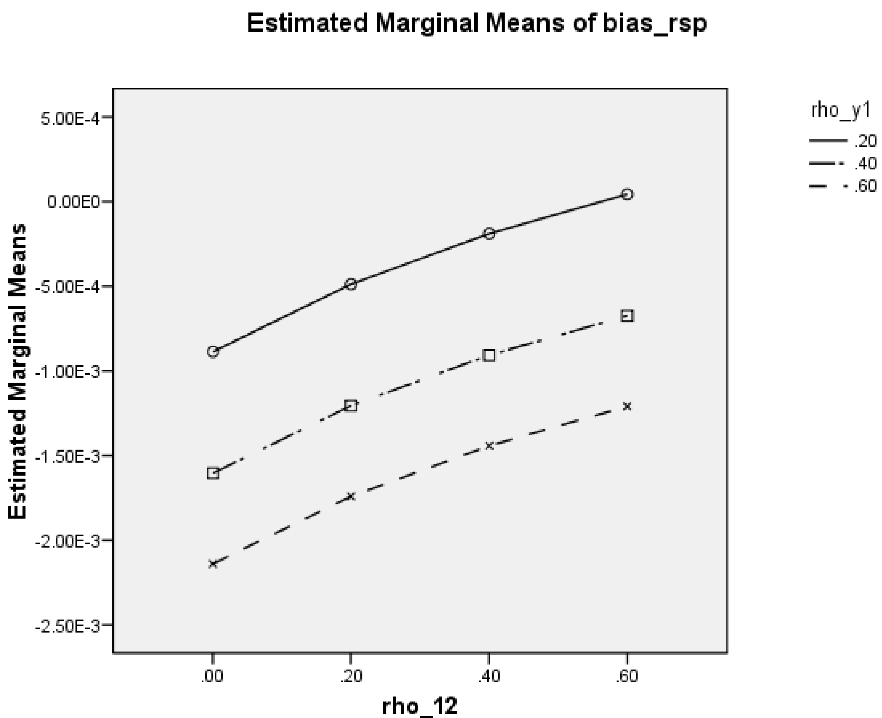

Figure A.5. Average Bias of r_{sp} for Two Independent Variables (n = 200, by ρ_{Y1})

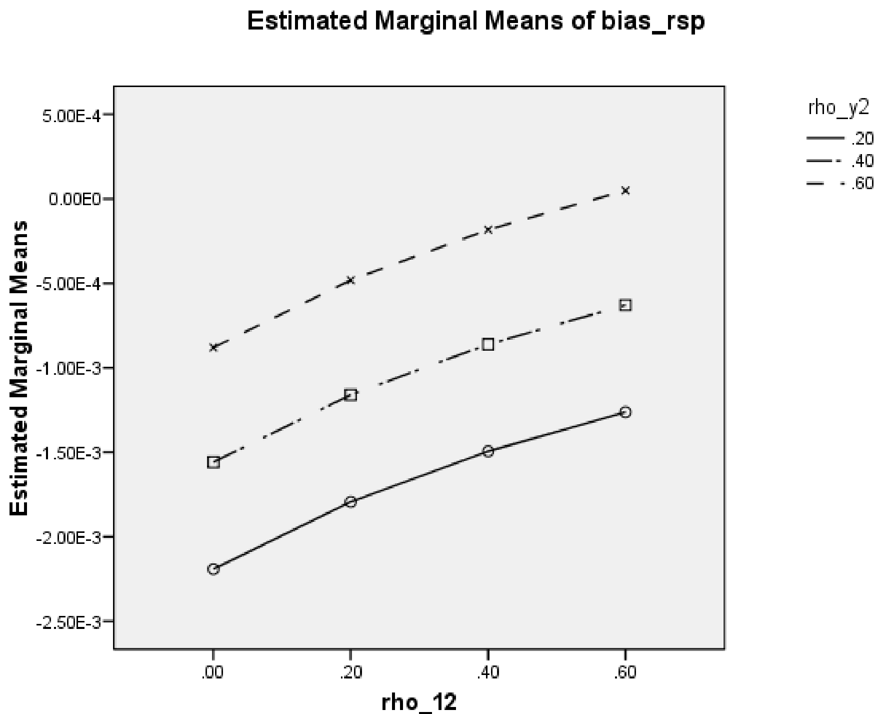

Figure A.6. Average Bias of r_{sp} for Two Independent Variables (n = 200, by ρ_{Y2})

Table A.4. Analysis of Variance for Differences between var(r_{sp}) and var(ρ_{sp}) for n = 200, Two Predictors

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	8.37E-005(a)	11	7.61E-006	101.445	.000
Intercept	.000	1	.000	1758.761	.000
ρ_{Y1}	5.95E-005	2	2.97E-005	396.300	.000
ρ_{Y2}	3.54E-006	2	1.77E-006	23.566	.000
ρ_{12}	2.57E-006	3	8.56E-007	11.417	.000
$\rho_{Y1} * \rho_{Y2}$	1.61E-005	4	4.02E-006	53.602	.000
Error	.010	134988	7.50E-008		
Total	.010	135000			
Corrected Total	.010	134999			

a R Squared = .008 (Adjusted R Squared = .008)

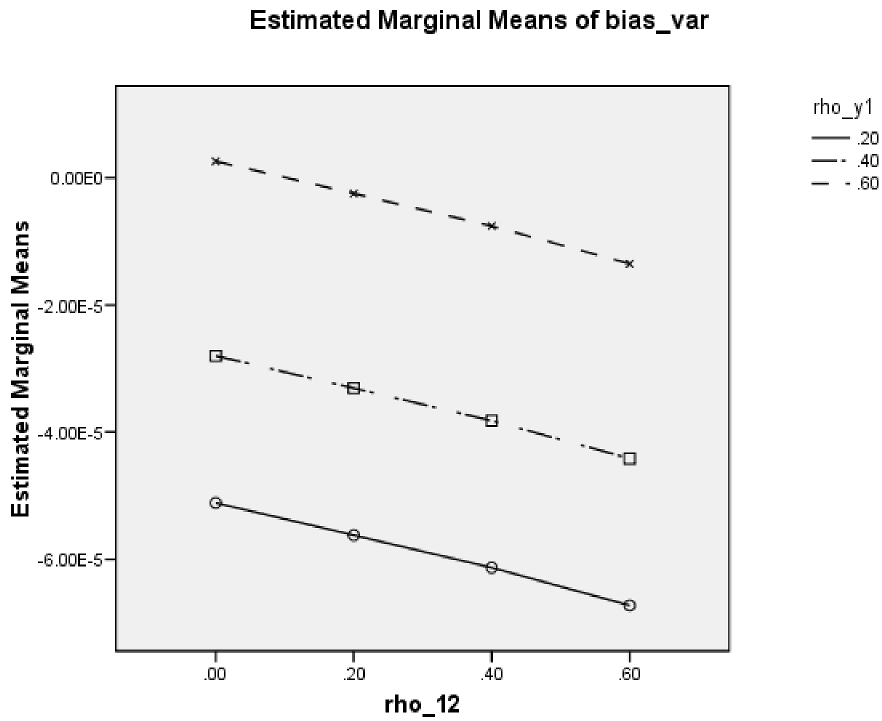

Figure A.7. Average Bias of var(r_{sp}) for Two Independent Variables (n = 200, by ρ_{Y1})

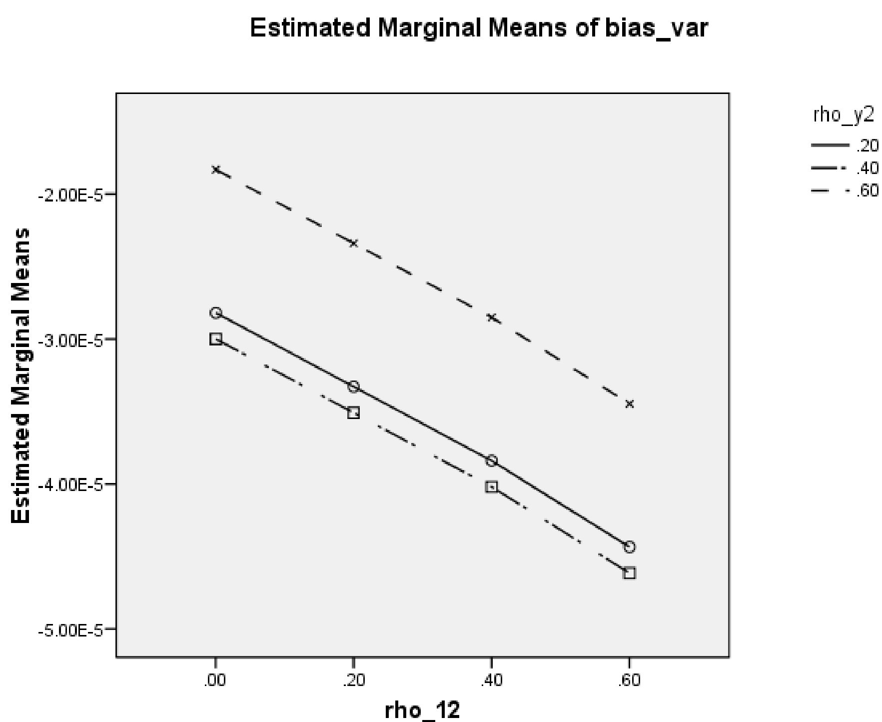

Figure A.8. Average Bias of var(r_{sp}) for Two Independent Variables (n = 200, by ρ_{Y2})

Table A.5. Analysis of Variance for Differences between r_{sp} and ρ_{sp} for n = 400, Two Predictors

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	.012(a)	7	.002	1.075	.377
Intercept	.046	1	.046	27.946	.000
ρ_{Y1}	.009	2	.004	2.741	.065
ρ_{Y2}	.002	2	.001	.522	.593
ρ_{12}	.003	3	.001	.532	.660
Error	220.879	134992	.002		
Total	220.942	135000			
Corrected Total	220.892	134999			

a R Squared = .000 (Adjusted R Squared = .000)

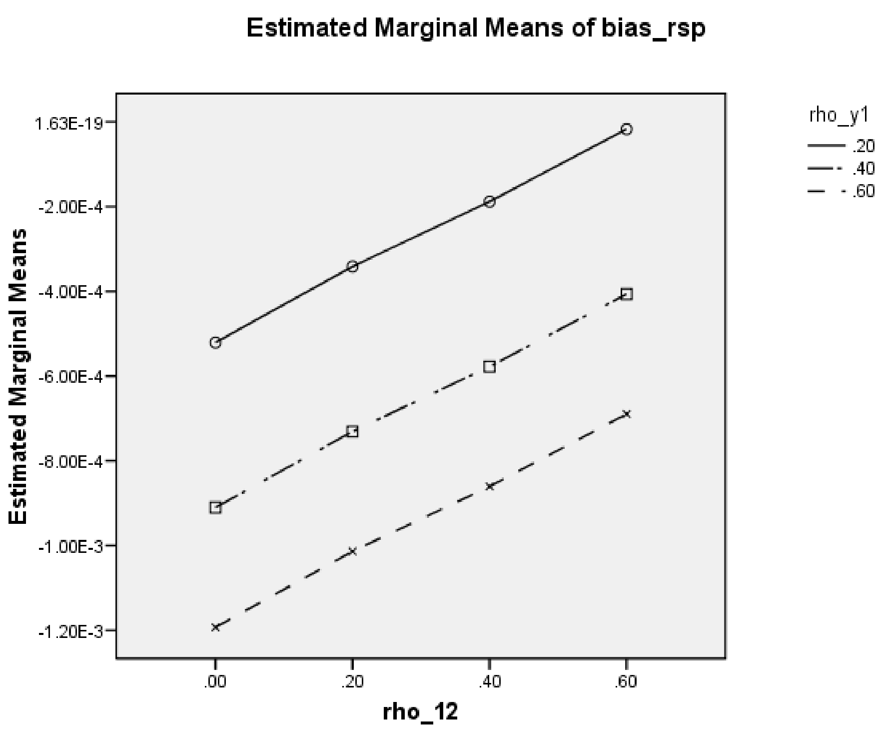

Figure A.9. Average Bias of r_{sp} for Two Independent Variables (n = 400, by ρ_{Y1})

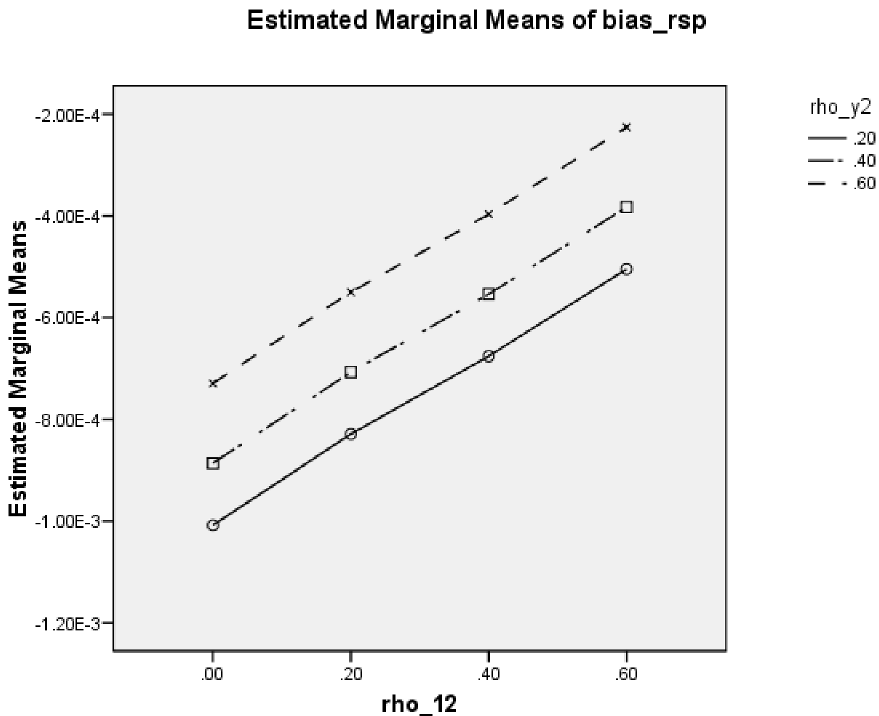

Figure A.10. Average Bias of r_{sp} for Two Independent Variables (n = 400, by ρ_{Y2})

Table A.6. Analysis of Variance for Differences between var(r_{sp}) and var(ρ_{sp}) for n = 400, Two Predictors

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	5.60E-006(a)	11	5.09E-007	53.080	.000
Intercept	9.40E-006	1	9.40E-006	978.947	.000
ρ_{Y1}	4.13E-006	2	2.07E-006	215.320	.000
ρ_{Y2}	2.14E-007	2	1.07E-007	11.148	.000
ρ_{12}	1.81E-007	3	6.04E-008	6.297	.000
$\rho_{Y1} * \rho_{Y2}$	9.56E-007	4	2.39E-007	24.896	.000
Error	.001	134988	9.60E-009		
Total	.001	135000			
Corrected Total	.001	134999			

a R Squared = .004 (Adjusted R Squared = .004)

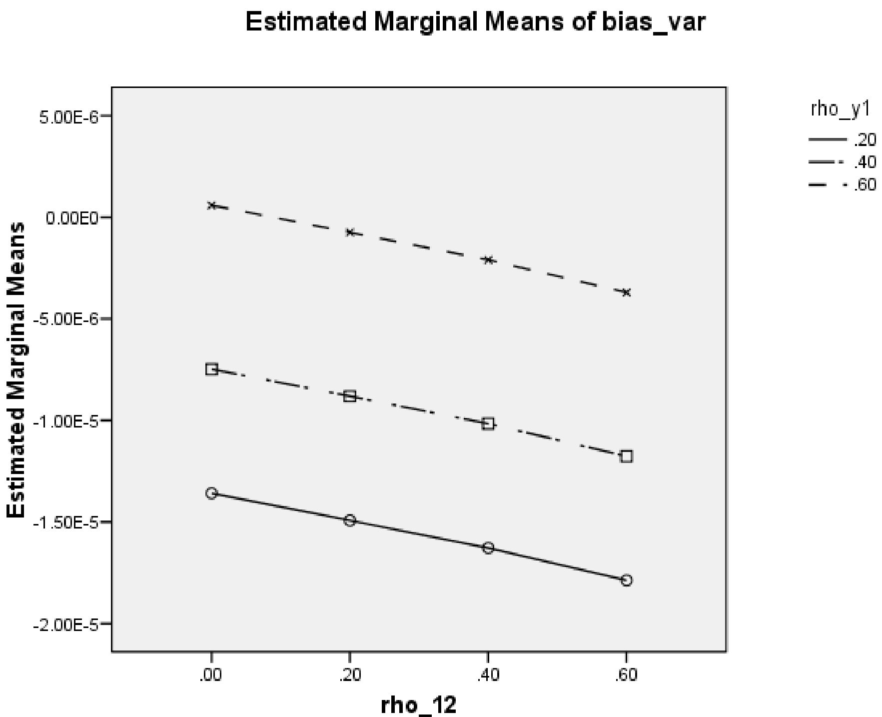

Figure A.11. Average Bias of var(r_{sp}) for Two Independent Variables (n = 400, by ρ_{Y1})

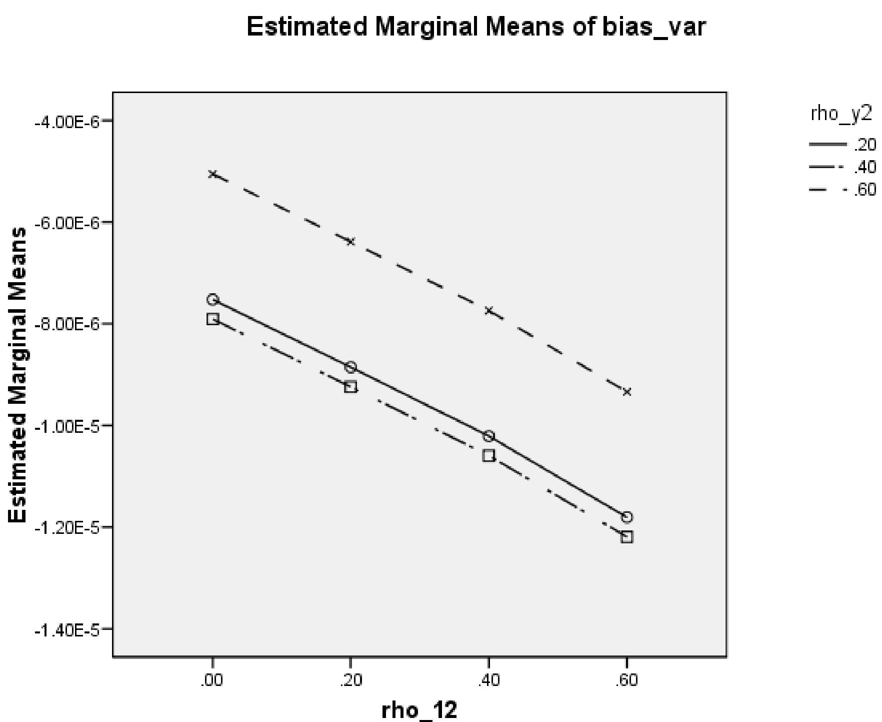

Figure A.12. Average Bias of var(r_{sp}) for Two Independent Variables (n = 400, by ρ_{Y2})

Table A.7. Analysis of Variance for Differences between r_{sp} and ρ_{sp} for n = 800, Two Predictors

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	.002(a)	7	.000	.341	.936
Intercept	.034	1	.034	42.938	.000
ρ_{Y1}	.001	2	.001	.754	.470
ρ_{Y2}	.000	2	.000	.284	.752
ρ_{12}	.000	3	.000	.151	.929
Error	105.454	134992	.001		
Total	105.494	135000			
Corrected Total	105.456	134999			

a R Squared = .000 (Adjusted R Squared = .000)

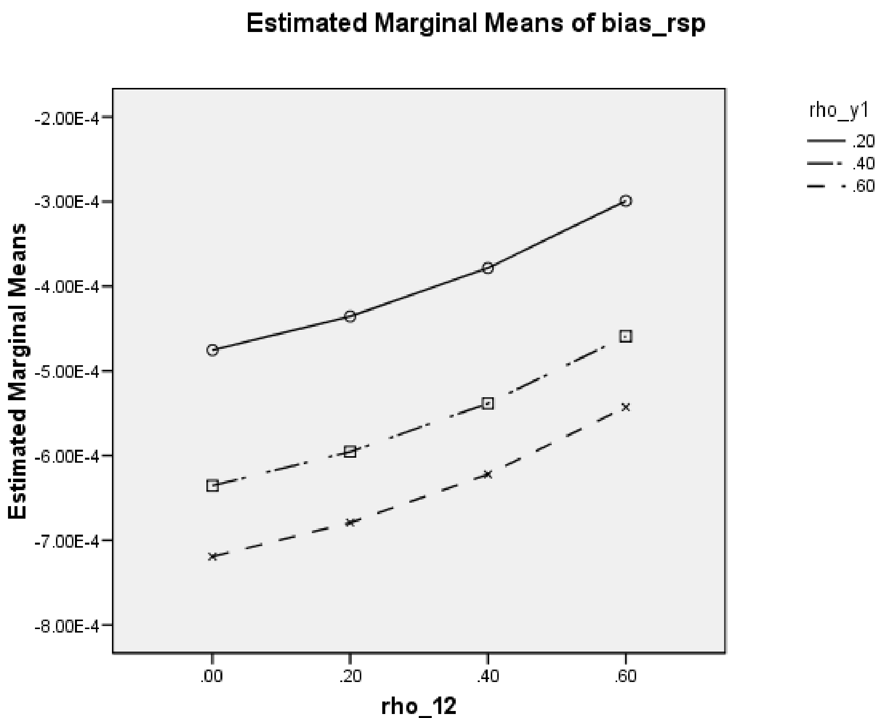

Figure A.13. Average Bias of r_{sp} for Two Independent Variables (n = 800, by ρ_{Y1})

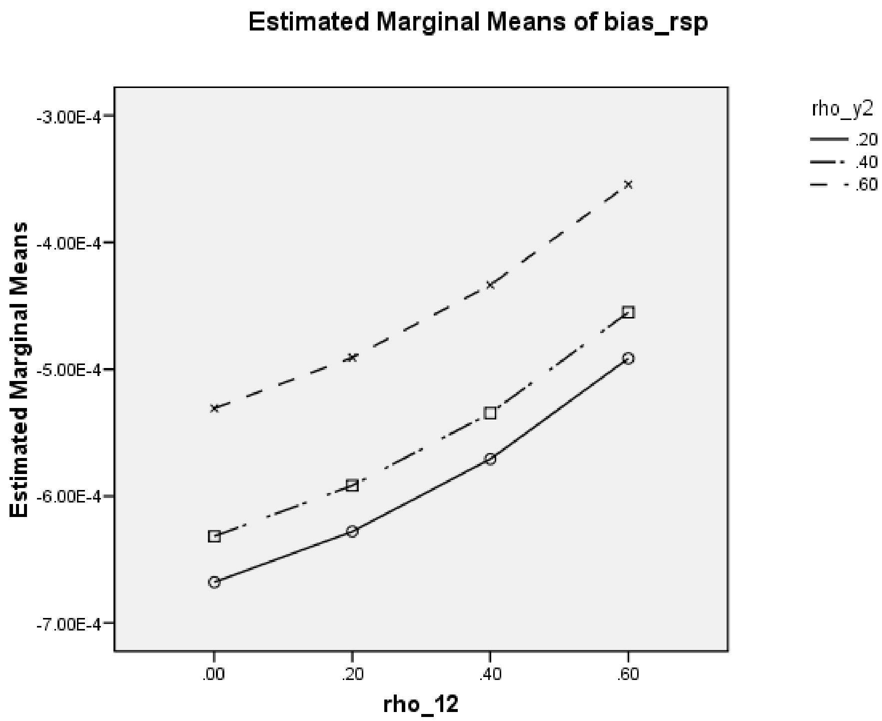

Figure A.14. Average Bias of r_{sp} for Two Independent Variables (n = 800, by ρ_{Y2})

Table A.8. Analysis of Variance for Differences between var(r_{sp}) and var(ρ_{sp}) for n = 800, Two Predictors

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	3.72E-007(a)	11	3.39E-008	28.561	.000
Intercept	5.64E-007	1	5.64E-007	475.427	.000
ρ_{Y1}	2.86E-007	2	1.43E-007	120.554	.000
ρ_{Y2}	7.09E-009	2	3.55E-009	2.992	.050
ρ_{12}	1.12E-008	3	3.74E-009	3.159	.024
$\rho_{Y1} * \rho_{Y2}$	5.83E-008	4	1.46E-008	12.291	.000
Error	.000	134988	1.19E-009		
Total	.000	135000			
Corrected Total	.000	134999			

a R Squared = .002 (Adjusted R Squared = .002)

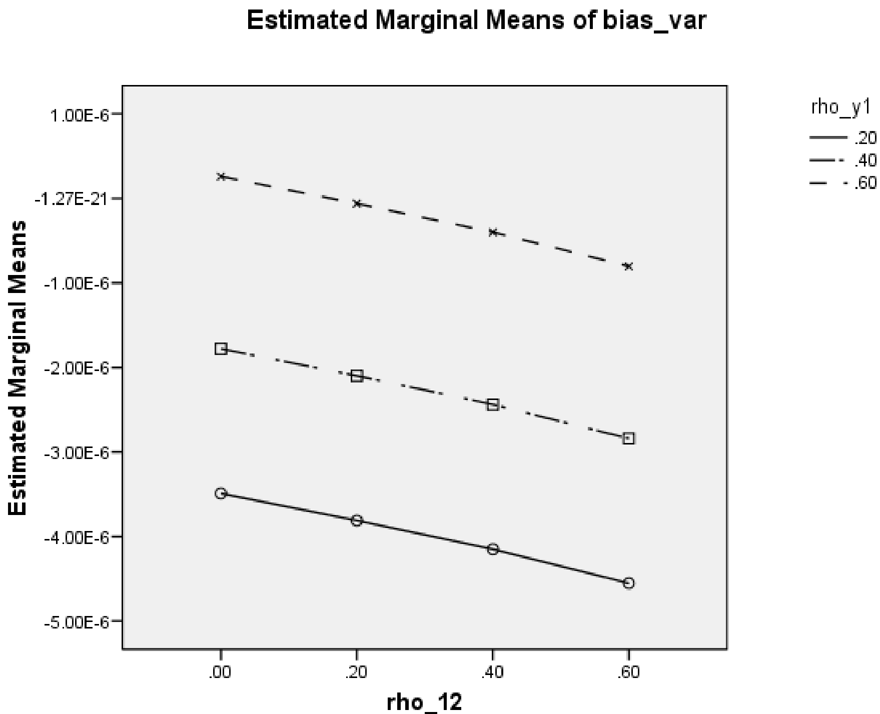

Figure A.15. Average Bias of var(r_{sp}) for Two Independent Variables (n = 800, by ρ_{Y1})

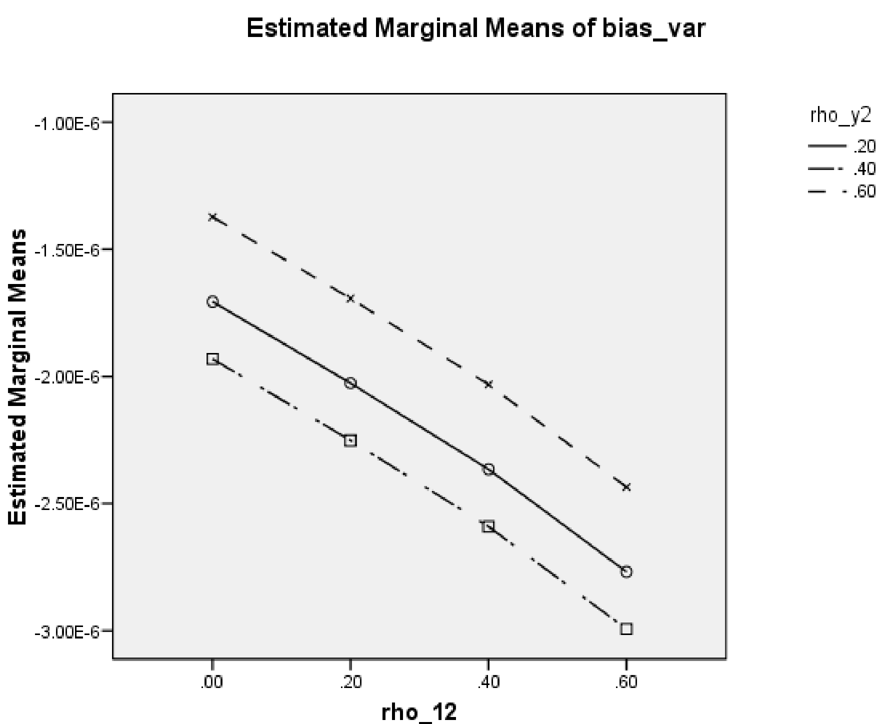

Figure A.16. Average Bias of var(r_{sp}) for Two Independent Variables (n = 800, by ρ_{Y2})

Table A.9. Analysis of Variance for Differences between r_{sp} and ρ_{sp} for n = 100, Three Predictors

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	.096(a)	6	.016	1.929	.072
Intercept	.199	1	.199	24.063	.000
ρ_{Y1}	.056	1	.056	6.827	.009
ρ_{12}	.026	3	.009	1.066	.362
ρ_{13}	.000	1	.000	.028	.867
ρ_{23}	.011	1	.011	1.347	.246
Error	495.532	59993	.008		
Total	495.977	60000			
Corrected Total	495.627	59999			

a R Squared = .000 (Adjusted R Squared = .000)

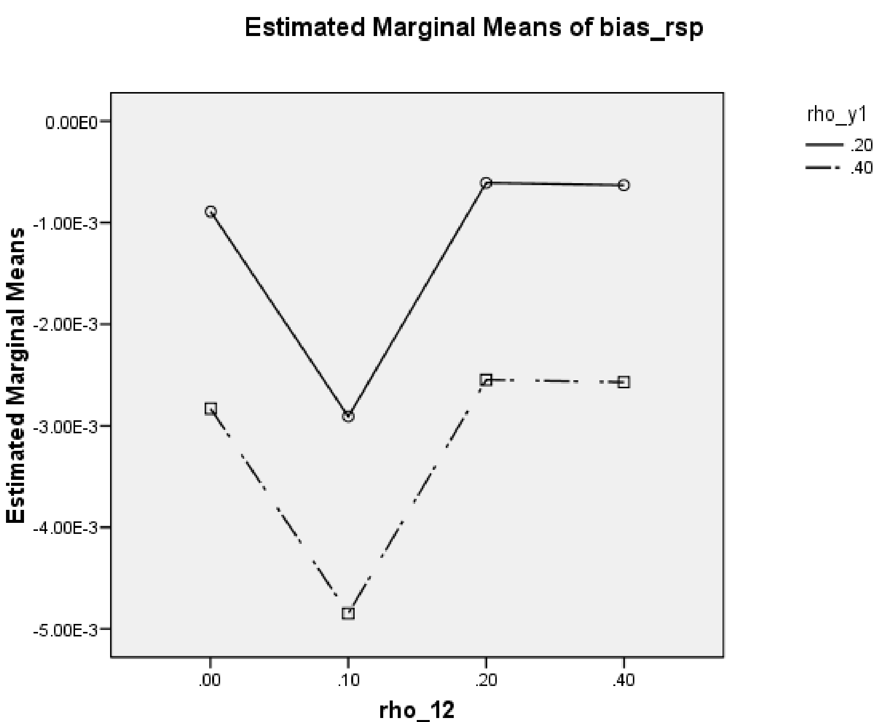

Figure A.17. Average Bias of r_{sp} for Three Independent Variables (n = 100, by ρ_{Y1})

Table A.10. Analysis of Variance for Differences between var(r_{sp}) and var(ρ_{sp}) for n = 100, Three Predictors

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	.015(a)	10	.002	2050.034	.000
Intercept	.008	1	.008	10088.936	.000
ρ_{Y1}	.011	1	.011	15215.909	.000
ρ_{12}	.000	3	.000	133.160	.000
ρ_{13}	1.73E-005	1	1.73E-005	23.041	.000
ρ_{23}	1.55E-006	1	1.55E-006	2.062	.151
$\rho_{Y1}*\rho_{12}$	2.00E-005	3	6.66E-006	8.870	.000
$\rho_{Y1}*\rho_{23}$	1.32E-005	1	1.32E-005	17.627	.000
Error	.045	59989	7.51E-007		
Total	.071	60000			
Corrected Total	.060	59999			

a R Squared = .255 (Adjusted R Squared = .255)

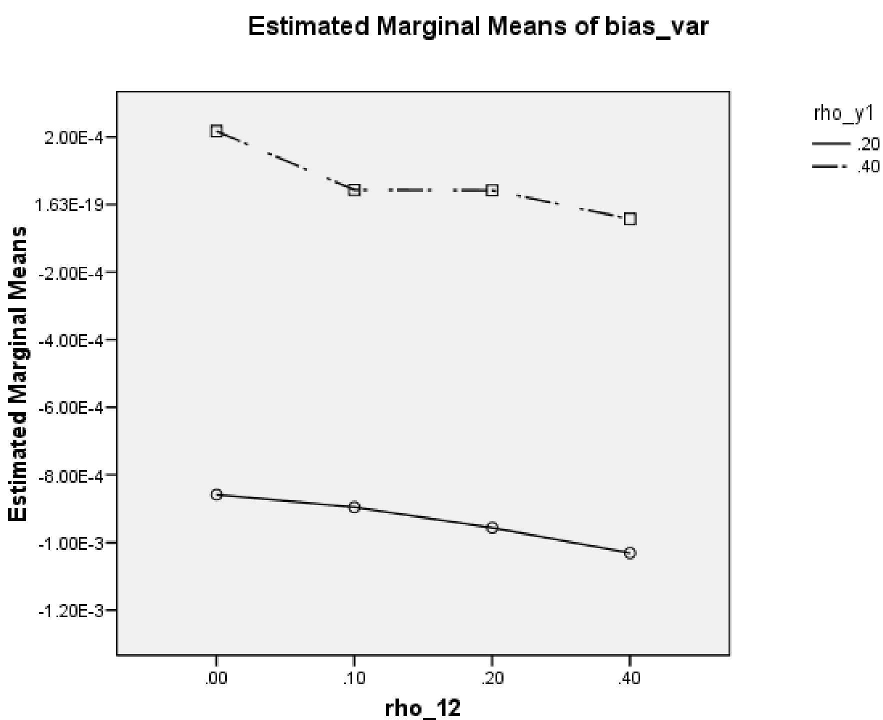

Figure A.18. Average Bias of var(r_{sp}) for Three Independent Variables (n = 100, by ρ_{Y1} with ρ_{12})

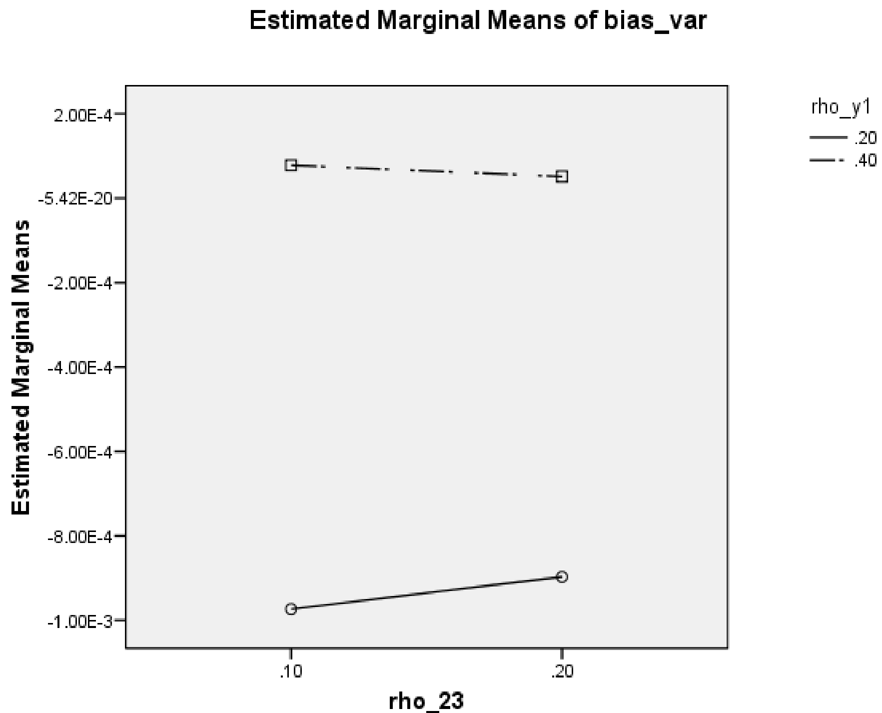

Figure A.19. Average Bias of var(r_{sp}) for Three Independent Variables (n = 100, by ρ_{Y1} with ρ_{23})

Table A.11. Analysis of Variance for Differences between r_{sp} and ρ_{sp} for n = 200, Three Predictors

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	.033(a)	6	.005	1.333	.238
Intercept	.011	1	.011	2.587	.108
ρ_{Y1}	.027	1	.027	6.563	.010
ρ_{12}	.004	3	.001	.313	.816
ρ_{13}	7.57E-005	1	7.57E-005	.019	.892
ρ_{23}	3.31E-006	1	3.31E-006	.001	.977
Error	244.982	59993	.004		
Total	245.030	60000			
Corrected Total	245.014	59999			

a R Squared = .000 (Adjusted R Squared = .000)

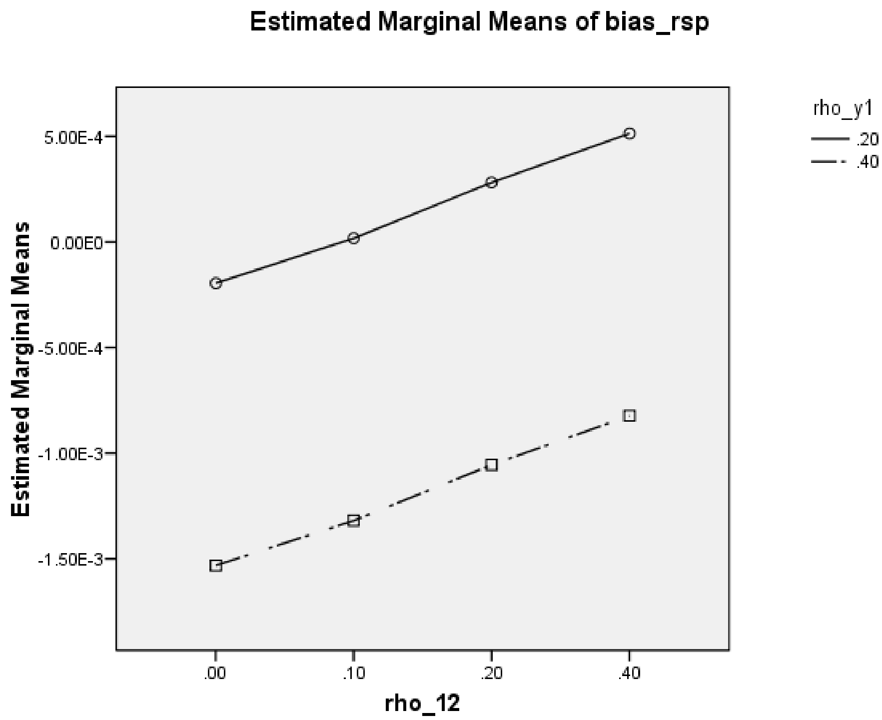

Figure A.20. Average Bias of r_{sp} for Three Independent Variables (n = 200, by ρ_{Y1} with ρ_{12})

Table A.12. Analysis of Variance for Differences between var(r_{sp}) and var(ρ_{sp}) for n = 200, Three Predictors

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	.003(a)	9	.000	3896.575	.000
Intercept	.000	1	.000	4828.978	.000
ρ_{Y1}	.003	1	.003	29501.603	.000
ρ_{12}	8.59E-005	3	2.86E-005	293.048	.000
ρ_{13}	3.85E-006	1	3.85E-006	39.366	.000
ρ_{23}	3.00E-006	1	3.00E-006	30.670	.000
$\rho_{Y1}*\rho_{12}$	4.15E-006	3	1.38E-006	14.148	.000
Error	.006	59990	9.78E-008		
Total	.010	60000			
Corrected Total	.009	59999			

a R Squared = .369 (Adjusted R Squared = .369)

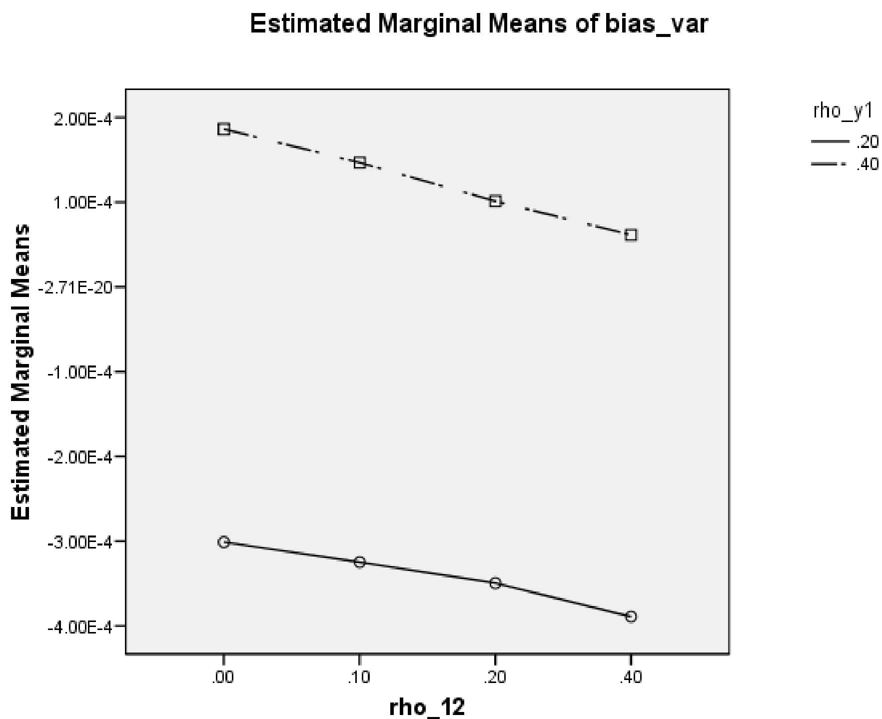

Figure A.21. Average Bias of var(r_{sp}) for Three Independent Variables (n = 200, by ρ_{Y1} with ρ_{12})

Table A.13. Analysis of Variance for Differences between r_{sp} and ρ_{sp} for n = 400, Three Predictors

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	.007(a)	6	.001	.561	.762
Intercept	.012	1	.012	5.947	.015
ρ_{Y1}	.004	1	.004	2.161	.142
ρ_{12}	.002	3	.001	.274	.844
ρ_{13}	1.88E-005	1	1.88E-005	.009	.924
ρ_{23}	6.25E-006	1	6.25E-006	.003	.956
Error	123.889	59993	.002		
Total	123.914	60000			
Corrected Total	123.896	59999			

a R Squared = .000 (Adjusted R Squared = .000)

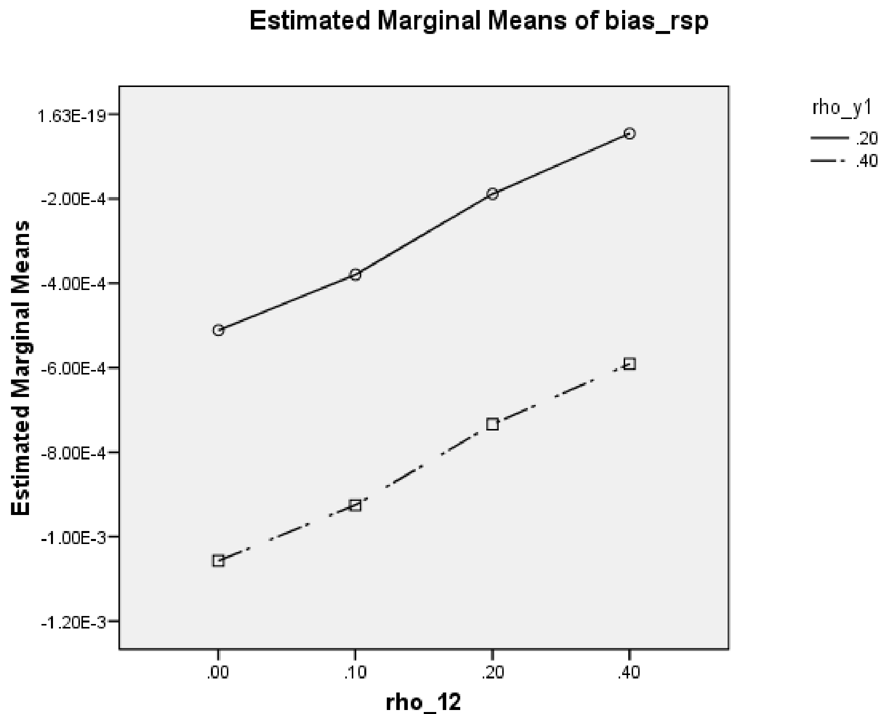

Figure A.22. Average Bias of r_{sp} for Three Independent Variables (n = 400, by ρ_{Y1} with ρ_{12})

Table A.14. Analysis of Variance for Differences between var(r_{sp}) and var(ρ_{sp}) for n = 400, Three Predictors

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	.001(a)	9	9.08E-005	7624.142	.000
Intercept	2.18E-005	1	2.18E-005	1831.987	.000
ρ_{Y1}	.001	1	.001	57759.388	.000
ρ_{12}	1.99E-005	3	6.65E-006	557.984	.000
ρ_{13}	8.98E-007	1	8.98E-007	75.401	.000
ρ_{23}	7.96E-007	1	7.96E-007	66.812	.000
$\rho_{Y1}*\rho_{12}$	9.26E-007	3	3.09E-007	25.904	.000
Error	.001	59990	1.19E-008		
Total	.002	60000			
Corrected Total	.002	59999			

a R Squared = .534 (Adjusted R Squared = .533)

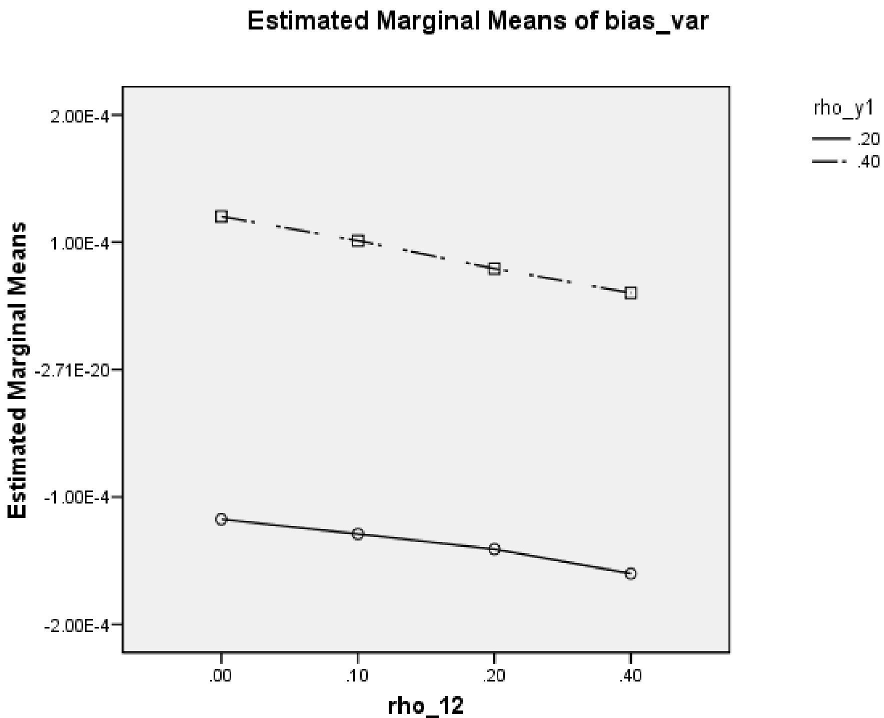

Figure A.23. Average Bias of var(r_{sp}) for Three Independent Variables (n = 400, by ρ_{Y1} with ρ_{23})

Table A.15. Analysis of Variance for Differences between r_{sp} and ρ_{sp} for n = 800, Three Predictors

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	.002(a)	6	.000	.369	.899
Intercept	.010	1	.010	9.760	.002
ρ_{Y1}	.002	1	.002	2.042	.153
ρ_{12}	7.65E-005	3	2.55E-005	.025	.995
ρ_{13}	.000	1	.000	.135	.714
ρ_{23}	3.16E-005	1	3.16E-005	.031	.860
Error	60.612	59993	.001		
Total	60.629	60000			
Corrected Total	60.614	59999			

a R Squared = .000 (Adjusted R Squared = .000)

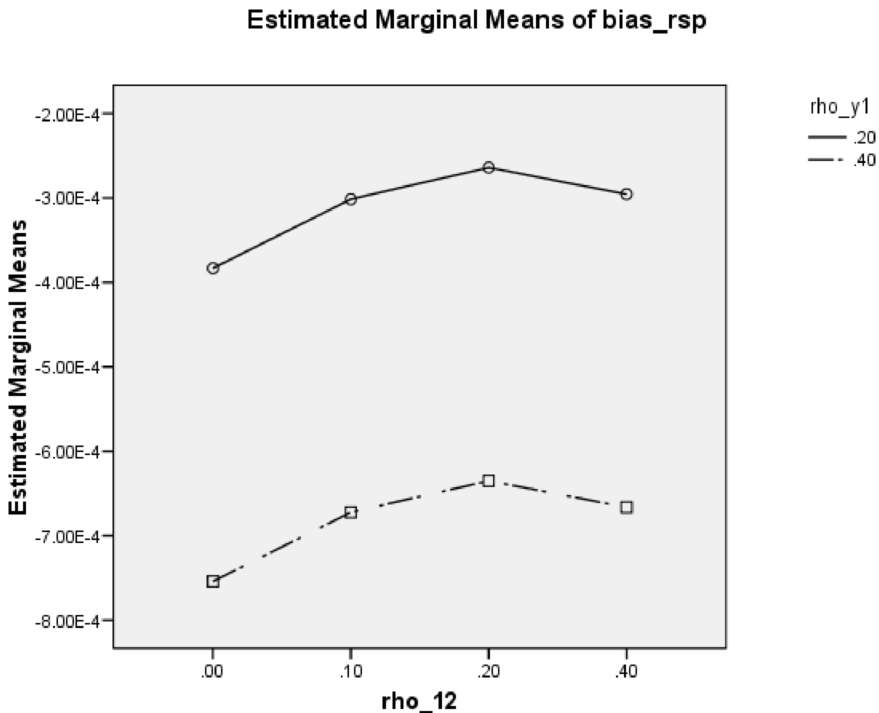

Figure A.24. Average Bias of r_{sp} for Three Independent Variables (n = 800, by ρ_{Y1} with ρ_{12})

Table A.16. Analysis of Variance for Differences between var(r_{sp}) and var(ρ_{sp}) for n = 800, Three Predictor

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	.000(a)	10	2.01E-005	13576.049	.000
Intercept	1.56E-006	1	1.56E-006	1055.629	.000
ρ_{Y1}	.000	1	.000	114307.135	.000
ρ_{12}	4.82E-006	3	1.61E-006	1086.637	.000
ρ_{13}	2.14E-007	1	2.14E-007	144.449	.000
ρ_{23}	2.09E-007	1	2.09E-007	141.082	.000
$\rho_{Y1}*\rho_{12}$	1.95E-007	3	6.51E-008	44.032	.000
$\rho_{Y1}*\rho_{13}$	1.32E-008	1	1.32E-008	8.915	.003
Error	8.87E-005	59989	1.48E-009		
Total	.000	60000			
Corrected Total	.000	59999			

a R Squared = .694 (Adjusted R Squared = .693)

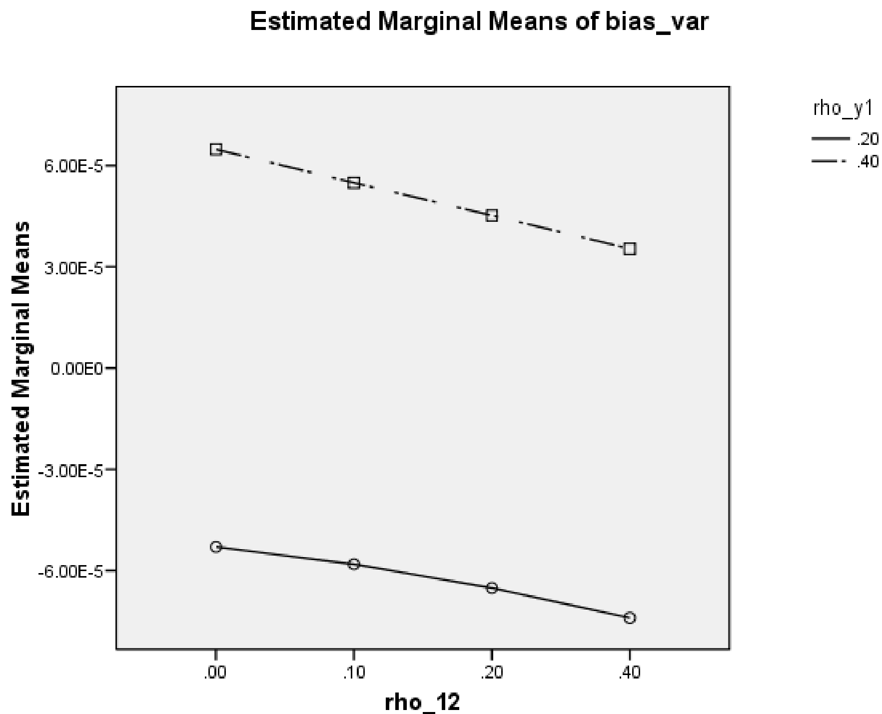

Figure A.25. Average Bias of var(r_{sp}) for Three Independent Variables (n = 800, by ρ_{Y1} with ρ_{12})

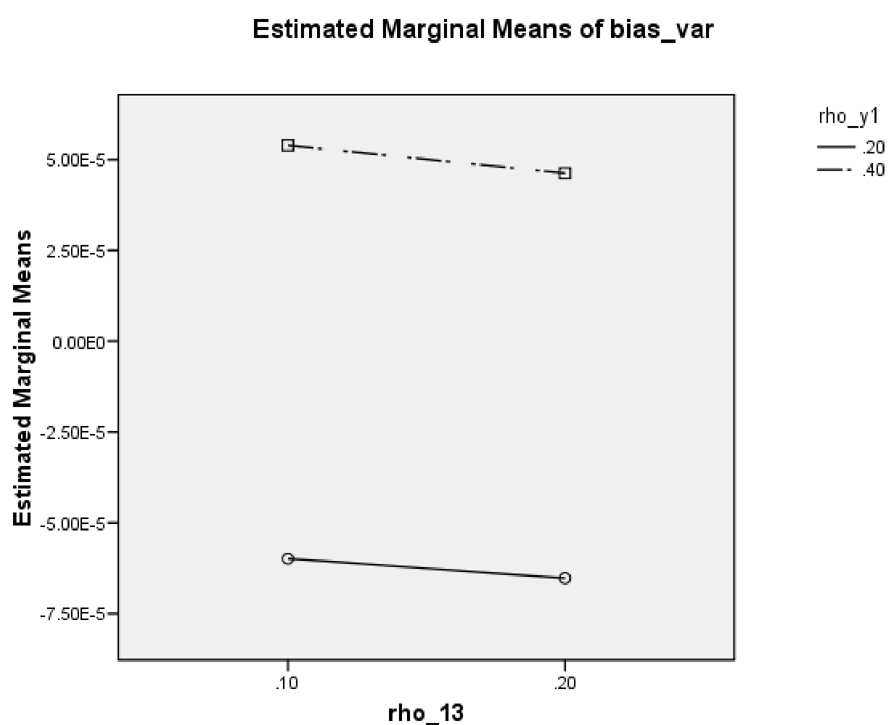

Figure A.26. Average Bias of var(r_{sp}) for Three Independent Variables (n = 800, by ρ_{Y1} with ρ_{13})

APPENDIX B

SIMULATION CODE

Simulation code for the two predictors model, condition 1

```
proc IML;

      kvec={50, 100, 200, 400, 800};

      Do ik=1 to 5;
      k=kvec[ik,];

      nseed=24687; * number of seed;
      nrep=5000; * nrep is number of replications;
      Z=j(k,3);
      Z1=j(k,3);
X0 = j(k,1,1);  * column of 1 for the one with no predictor;

            cor={1.0  0.2  0.2,
                 0.2  1.0  0.0,
                 0.2  0.o  1.0};

Do ireps=1 to nrep; *replication loop;
itotal = ireps+ik;
* these loops make up indep normal to eventually represent Y and X1 and
X2;
            Do i=1 to k;
                  Do j=1 to 3;
                    Z[i,j]=rannor(nseed);
                  end;
            end; * end of i loop;
*print ireps Z;
* next I make delta and X correlated, using the cholesky factor;
            cr=root(cor);  *Cholesky decomposition; *print cr;
            Z1=z*cr; *note I have matrix multiplication here, not
element * matrix;
            *print z  z1;
*I create different matrix with intercept here for the different R2;
      Xa = Z1[, {2 3}];
      Y = Z1[,1];
      X12 = x0||Xa; *intercept and predictor;
      X1 = X0||Z1[,2];
      X2 = X0||Z1[,3];
      X1_1 = Z1[,2];
      X2_2 = Z1[,3];

      namesh = { 'Z0' 'Z1' 'Z2'};
      if ireps=1 then  HDATA=Z1;
```

```
      if ireps>1 then HDATAr=Z1;
      if ireps>1 then hdata=hdata//hdatar;

****************************************;
*compute correlation among variables;
n=nrow(Z1);
SUM = Z1[+,]; pr=SUM`*SUM; *print pr;
zpr=Z1`*Z1; *print zpr;
XPX = (Z1`*Z1)-((SUM`*SUM)/n); *print xpx;
v = VECDIAG(XPX); *print v;
S = DIAG(1/SQRT(v));*print s;
CORR=S*XPX*S;
*PRINT CORR;

*************************************************;
*Model 1;
n=nrow(X1); * observations;
p = ncol(X1); *variables;
beta1 = inv(X1`*X1)*X1`*Y; *regression coeff; *print beta1;
e_1 = Y - X1*beta1; *vector of residuals;
df = n-p;
SSQ_1 = e_1`*e_1/df;  *estimator of MSE;
beta_v1 = inv(X1`*X1)*SSQ_1; *variance matrix for beta;
beta_sd1 = sqrt(vecdiag(beta_v1)); *standard error of beta;
t11 = beta1/beta_sd1; *t test for the slope; *print t1;
p_value1 = 1 - probf(t11#t11,1,df);* p-value for t tailed;

************************************************************;
      namesm1 = { 'beta0' 'beta1' 't00' 't11' };
      if ireps=1 then  m1DATA=beta1`||t11`;
      if ireps>1 then m1DATAr=beta1`||t11`;
      if ireps>1 then m1data=m1data//m1datar;

************************************************************;
*Model 2;
n=nrow(X2); * observations;
p = ncol(X2); *variables;
beta2 = inv(X2`*X2)*X2`*Y; *regression coeff; *print beta2;
e_2 = Y - X2*beta2; *vector of residuals;
df = n-p;
SSQ_2 = e_2`*e_2/df;  *estimator of MSE;
beta_v2 = inv(X2`*X2)*SSQ_2; *variance matrix for beta;
beta_sd2 = sqrt(vecdiag(beta_v2)); *standard error of beta;
t22 = beta2/beta_sd2; *t test for the slope; *print t2;
p_value2 = 1 - probf(t22#t22,1,df);* p-value for t tailed;

***********************************************************;
      namesm2 = { 'bet0' 'beta2' 'tt0' 't22' };
      if ireps=1 then  m2DATA=beta2`||t22`;
      if ireps>1 then m2DATAr=beta2`||t22`;
      if ireps>1 then m2data=m2data//m2datar;
**************************************************************;
* Full model;
n=nrow(X12); * observations;
p = ncol(X12); *variables;

beta12 = inv(X12`*X12)*X12`*Y; *regression coeff; *print beta12;
```

```
e_12 = Y - X12*beta12; *vector of residuals;

df = n-p;

SSQ_12 = e_12`*e_12/df;  *estimator of MSE;

beta_v12 = inv(X12`*X12)*SSQ_12; *variance matrix for beta;

beta_sd12 = sqrt(vecdiag(beta_v12)); *standard error of beta;

t12 = beta12/beta_sd12; *t test for the slope; *print t12;

p_value1 = 1 - probf(t12#t12,1,df);* p-value for t tailed;

*******************************************************;
beta12=beta12`; t12=t12`;
      namesm12 = { 'k' 'b0' 'b1' 'b2' 't0' 't1' 't2'};
      if itotal=2 then  m12DATA=k||beta12||t12;
      if itotal>2 then m12DATAr=k||beta12||t12;
      if itotal>2 then m12data=m12data//m12datar;

******************************************************;

***************************************************;
            BOLS_1 = INV(X1`*X1)*X1`*Y;   *Model with only x1;
            SST_1 = Y`*Y;
            SSR_1 = BOLS_1`*X1`*X1*BOLS_1;
            SSE_1 = SST_1-SSR_1;
            R2_1 = SSR_1/SST_1;
            Y_bar2 = (sum(Y)/n)**2;

*********************************************************;
            BOLS_2 = INV(X2`*X2)*X2`*Y;   *Model with only x2;
            SST_2 = Y`*Y;
            SSR_2 = BOLS_2`*X2`*X2*BOLS_2;
            SSE_2 = SST_2-SSR_2;
            R2_2 = SSR_2/SST_2;
                  *R2_2 = (SSR_2-Y_bar2)/(SST_2-Y_bar2);

            BOLS_12 = INV(X12`*X12)*X12`*Y;     *Model with only x1 and
x2;
            SST_12 = Y`*Y;
            SSR_12 = BOLS_12`*X12`*X12*BOLS_12;
            SSE_12 = SST_12-SSR_12;
            R2_12 = SSR_12/SST_12;

**********************************************************;
rsp1=j(k,1); rsp1_1=j(k,1); rsp2=j(k,1); rsp2_2=j(k,1);

      rsp1 = sqrt(R2_12 - R2_2);
      rsp1_1 = t12[,2]*(sqrt(1-R2_12))/sqrt(n - p - 1);

            if t12[,2] <0 then do;
            rsp1=rsp1*-1; end;

      rsp2 = sqrt(R2_12 - R2_1);
```

```
      rsp2_2 = t12[,3]*(sqrt(1-R2_12))/sqrt(n - p - 1);

            if t12[,3] <0 then do;
            rsp2=rsp2*-1; end;
**********************************************************;

R2_df1 = R2_12-(rsp1**2);
Rst = sqrt(R2_df1)/sqrt(R2_12);

rp_1 = sqrt(R2_12); a = sqrt(R2_12);
rp1_1= sqrt(R2_df1); b1 = sqrt(R2_df1);

part_01 = 1/(n*((rp_1**2)-(rp1_1**2)));
part01 = 1/(n*((a**2)-(b1**2)));

part_a1 = (rp_1**2)*(1-(rp_1**2))**2;
parta1= (a**2)*(1-(a**2))**2;

part_b1 = (rp1_1**2)*(1-(rp1_1**2))**2;
partb1 = (b1**2)*(1-b1**2)**2;

part_c1 = -2*rp_1*rp1_1*(((2*Rst-rp_1*rp1_1)*(1-(rp_1**2)-(rp1_1**2)-
(Rst**2))/2) + Rst**3);
partc1 = -2*a*b1*(((2*(b1/a)-a*b1)*(1-(a**2)-(b1**2)-
((b1/a)**2))/2)+(b1/a)**3);

var_rsp1 = (part_a1+part_b1+part_c1)*part_01;
varrsp1 = (parta1+partb1+partc1)*part01;
var_new1 = (R2_12**2 - 2*R2_12 + R2_df1 + 1 - R2_df1**2)/n;

**********************************************************;
R2_df2 = R2_12-(rsp2**2);
Rst2 = sqrt(R2_df2)/sqrt(R2_12);

rp_2 = sqrt(R2_12);
r_p1_2= sqrt(R2_df2); b2 = sqrt(R2_df2);

part_02 = 1/(n*((rp_2**2)-(r_p1_2**2)));
part02 = 1/(n*((a**2)-(b2**2)));

part_a2 = (rp_2**2)*(1-(rp_2**2))**2;
parta2= (a**2)*(1-(a**2))**2;

part_b2 = (r_p1_2**2)*(1-(r_p1_2**2))**2;
partb2 = (b2**2)*(1-b2**2)**2;

part_c2 = -2*rp_2*r_p1_2*(((2*Rst2-rp_2*r_p1_2)*(1-(rp_2**2)-
(r_p1_2**2)-(Rst2**2))/2) + Rst2**3);
partc2 = -2*a*b2*(((2*(b2/a)-a*b2)*(1-(a**2)-(b2**2)-
((b2/a)**2))/2)+(b2/a)**3);

var_rsp2 = (part_a2+part_b2+part_c2)*part_02;
varrsp2 = (parta2+partb2+partc2)*part02;

var_new2 = (R2_12**2 - 2*R2_12 + R2_df2 + 1 - R2_df2**2)/n;

***********************************************************;
```

```
      namesR2 = { 'k' 'R2_12' 'R2_1' 'R2_2' 'rsp1' 'rsp2' 'rsp1_1'
'rsp2_2' 'var_rsp1' 'var_rsp2'};
      if itotal=2 then
R2DATA=k||R2_12||R2_1||R2_2||rsp1||rsp2||rsp1_1||rsp2_2||var_rsp1||var_
rsp2;
      if itotal>2 then
R2DATAr=k||R2_12||R2_1||R2_2||rsp1||rsp2||rsp1_1||rsp2_2||var_rsp1||var
_rsp2;
      if itotal>2 then R2data=R2data//R2datar;

end;end;
create data from HDATA [colname=namesh];   append from hDATA; *end;
create m1 from m1DATA [colname=namesm1];   append from m1DATA; *end;
create m2 from m2DATA [colname=namesm2];   append from m2DATA; *end;
create m12 from m12DATA [colname=namesm12];   append from m12DATA;
*end;
create R2 from R2DATA [colname=namesR2];   append from R2DATA; *end;
```

REFERENCES

Aloe, A. M., & Becker, B. J. (2008). Teachers' verbal ability and students' academic achievement: Where is the evidence? (*Manuscript under review*).

Alf, E. F., & Graf, R. G. (1999). Asymptotic confidence limits for the difference between two squared multiple correlations: A simplified approach. *Psychological Methods, 4,* 70–75.

Bamberg, S., & Möser, G. (2006). Twenty years after Hines, Hungerford, and Tomera: A new meta-analysis of psycho-social determinants of pro-environmental behaviour. *Journal of Experimental Psychology*, *27*, 14-25.

Becker, B. J., & Wu, M.J. (2007). The synthesis of regression slopes in meta-analysis. *Statistical Science, 22*(3), 414-429.

Card, N. A., & Little, T. D. (2006). Proactive and reactive aggression in childhood and adolescence: A meta-analysis of differential relations with psychological adjustment. *International Journal of Behavioral Developmen*t, *30*(5), 466-480.

Chaney, B. (1995). *Student outcomes and the professional preparation of eighth-grade teachers in science and mathematics: NSF/NELS:88 Teacher transcript analysis*. Rockville, MD: Westat, Inc.

Cooper, H. M., & Hedges, L. V. (1994a). Potentials and limitations of research synthesis. In H. M. Cooper & L. V. Hedges (Eds.), *The handbook of research synthesis* (pp. 521-529). New York: Russell Sage Foundation.

Cooper, H. M., & Hedges, L. V. (Eds.) (1994b). *The handbook of research synthesis*. New York: Russell Sage Foundation.

Haring-Hidore, M., Stock, W.A., Okun, M. A., & Witter, R. A. (1985). Marital status and subjective well-being: A research synthesis. *Journal of Marriage and the Family*, *47*(4), 947-953.

Hedges, L. V. (1994). Fixed effects models. In H. M. Cooper & L. V. Hedges (Eds.), *The handbook of research synthesis* (pp. 521-529). New York: Russell Sage Foundation.

Hedges, L. V., & Olkin, I. (1981). The asymptotic distribution of commonality components. *Psychometrika, 46*, 331-336.

Hedges, L. V., & Olkin, I. (1985). *Statistical methods for meta-analysis*. Orlando, FL: Academic Press.

Hedges, L. V., & Vevea, J. L. (1998). Fixed- and random-effects models in meta-analysis. *Psychological Methods, 3*(4), 486-504.

Kerlinger, F. N. (1979). *Behavioral research: A conceptual approach*. New York: Holt, Rinehart and Winston.

Keef, S. P., & Roberts, L. A. (2004). The meta-analysis of partial effect sizes. *British Journal of Mathematical and Statistical Psychology, 57*(1), 97-129.

Koricheva, J., Nykänen, H., & Gianoli, E. (2004). Meta-analysis of trade-offs among plant antiherbivore defenses: Are plants jacks-of-all-trades, masters of all? *The American Naturalist*, *163*(4), E64-E75.

McCartney, K., & Rosenthal, R. (2000). Effect size, practical importance, and social policy for children. *Child Development, 71*, 173-180.

Meehl, P. E. (1990). Why summaries of research on psychological theories are often uninterpretable. *Psychological Reports, 66*, 195-244.

Monk, D. H. (1994). Subject area preparation of secondary mathematics and science teachers and student achievement. *Economics of Education Review, 13*(2), 125-145.

Olkin, I. & Finn, J. D. (1995). Correlations redux. *Psychological Bulletin, 118* (1), 155-164.

Pedhazur, E. J. (1997). *Multiple regression in behavioral research: Explanation and prediction* (3rd ed.). Orlando, FL: Holt, Rinehart, & Winston.

Phaf, R. H., & Khan, K. (2006). The automaticity of emotional stroop: A meta-analysis. *Journal of Behavior Therapy and Experimental Psychiatry*, *38*, 184-199.

Rao, C. R. (1973). *Linear statistical inference and its applications* (2nd ed.). New York, NY: Wiley.

Stankowich, T. & Blumstein, D. T. (2005). Fear in animals: A meta-analysis and review of risk assessment. *Proceedings: Biological Science*, 272, 2627-2643.

Stanley, T. D., & Jarrell, S.B. (1989). Meta-regression analysis: A quantitative method of literature surveys. *Journal of Economic Surveys, 3*(2), 161-170.

Stanley, T. D., & Jarrell, S.B. (2005). Meta-regression analysis: A quantitative method of literature surveys. *Journal of Economic Surveys, 19*(3), 299-308. [Reprinted version of the 1989 paper].

Timm, N. H. (2004). Estimating effect sizes in exploratory experimental studies when using a linear model. *The American Statistician, 58*(3), 213-217.

Welten, D. C., Kemper, H.C. G., Post, G. B., & Van Staveren, W. A. (1995). A meta-analysis of the effect of calcium intake on bone mass in young and middle aged females and males. *Journ*

BIOGRAPHICAL SKETCH

Ariel M. Aloe was born in Buenos Aires, Argentina on March 16, 1975. He received a Bachelor of Physical Education from Universidad de Flores in 2001. He received a Master in Educational Psychology from Loyola University Chicago in 2005 and a Master of Science in Statistics from Florida State University in 2008. While he pursued his doctoral degree in Measurement and Statistics at Florida State University, he worked as a research assistant for Professor Betsy J. Becker. His research interests are meta-analysis and teacher's qualification.