THE FLORIDA STATE UNIVERSITY

COLLEGE OF ENGINEERING

PARAMETRIC OPTIMIZATION OF STEEL FLOOR SYSTEM COST USING EVOLVER

By

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A Thesis submitted to the
Department of Civil and Environmental Engineering
in partial fulfillment of the
requirements for the degree of
Master of Science

Degree Awarded:
Summer Semester, 2006

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ACKNOWLEDGEMENTS

I would like to express my appreciation to everyone who has made this thesis possible. Thank you to my Thesis Director, Dr. Primus Mtenga, for his guidance, advice, and encouragement. To Dr. Simon Thuo, Dr. John Sobanjo, and Dr. Kamal Tawfiq for participating on my Committee. To Francisco Ortiz and Dr. Amy Chan-Hilton for their help with genetic algorithm optimization. Finally, many thanks to the steel manufacturers and fabricators, including Blair Williams of Jackson-Cook, LC in Tallahassee, FL; John Montgomery Jr. of Southland Tube, Inc. in Birmingham, AL; Mark Perry of Quincy-Joist Company, Inc. in Quincy, FL; Harold Turner of Qualico Steel Company, Inc. in Webb, AL; Daniel Davis of Bell Steel Company in Pensacola, FL; Gordon Matlock of Steward Steel, Inc. in Sikeston, MO; Scott Tucker of Southern Steel Company, LLC in Charlotte, NC; and Samuel Sprouse of Barkley Engineers, Inc. in Tallahassee, FL who have provided the steel fabrication and cost estimate information necessary to complete this thesis.

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NOTATION

A	Cross-sectional area, in.^{2}
A_e	Effective area for steel, in.^{2}
A_g	Gross area for steel, in.^{2}
C	Compression force, kips
C	Cost, $
E	Modulus of elasticity for steel, ksi
F_{cr}	Critical stress for steel, ksi
F_u	Tensile strength of type of steel, ksi
F_y	Yield stress of type of steel, ksi
I	Moment of inertia, in.^{4}
K	Effective length factor for steel compression members
L	Length, ft
L_r	Roof live load, psf
M	Calculated moment, kip-ft
M_n	Nominal flexural strength for concrete, kip-ft
M_p	Plastic bending moment for steel, kip-ft
N	Internal force in truss member due to real loads applied to truss, kips
P	Concentrated load, kips
P_n	Nominal axial strength, kips
Q	Full reduction factor for slender steel compression sections
R	Flexural resistance factor for steel reinforcement
R_1, R_2	Reduction factors for roof live load
T	Tension force, kips

U	Shear lag coefficient
V	Shear force, kips
W_b	Weight of individual beam, kips
W_t	Weight of individual trusses, kips
Z	Plastic section modulus for steel, in.^{3}
b	Depth of steel compression section, in.
b	Width of concrete slab section, in.
b_e	Reduced effective depth for slender steel compression section, in.
c	Concrete cover for steel reinforcement, in.
c_b	Cost factor applied to beams
c_t	Cost factor applied to trusses
d	Effective depth of concrete slab, in.
d_b	Bar diameter of steel reinforcement, in.
d_n	Nominal depth of steel member, in.
f	Compressive stress in stiffened steel section, ksi
f'_c	Compressive strength of concrete, ksi
h, h_t	Height of truss girders, ft.
h	Thickness of concrete slab, in.
i	Number of each additional double-panel sets beyond initial 2-panel truss
l	Length, ft.
n	Virtual internal force in truss member due to unit load applied to truss, kips
n_b	Number of beams
n_t	Number of truss girders
q_{DL}	Uniformly distributed dead load per area, psf
q_{LL}	Uniformly distributed live load per area, psf
r	Radius of gyration, in.
s_b	Spacing of beams, ft.
s_t	Spacing of trusses, ft.
t	Thickness of concrete slab, in.
t	Thickness of element of steel section, in.
w	Uniformly distributed total load per length, plf

w_{DL}	Uniformly distributed dead load per length, plf
w_{LL}	Uniformly distributed live load per length, plf
x_1, x_2	Dimensions of floor system area, ft.
Δ	Deflection, in.
β_1	Factor relating to compressive strength of concrete
γ_c	Unit weight of concrete, pcf
ε_t	Net tensile strength for steel reinforcement
ε_u	Compression-controlled strain limit for steel reinforcement
λ	Slenderness parameter for steel
λ_c	Slenderness parameter for steel column
λ_p	Limiting slenderness parameter for compact section
λ_r	Limiting slenderness parameter for noncompact section
ρ	Steel reinforcement ratio
ϕ	Resistance factor for steel
ϕ	Strength reduction factor for concrete

ABSTRACT

This paper examines the application of *Evolver*, a genetic algorithm (GA) solving program, in a three-parameter optimization of a steel truss floor system with a concrete slab floor deck. The floor system is comprised of truss girders supporting beams running in a direction perpendicular to the truss girders with a composite floor deck along the top. Using Evolver, three parameters are optimized for two truss girder topologies in order to find the least cost floor system. The weight of the structural members is correlated to the expenses of material, labor, equipment, and overhead and profit required for the construction of the floor through information given by *Means Building Construction Cost Data* and interviews with steel fabricators. This procedure may be modified to optimize the cost of any floor area size that may use different truss girder topologies, beam sections, and connections.

Parametric optimization is defined in this paper as the combination of configuration, size, and topology optimization of a truss girder, the size optimization of beams, and the optimization of the spacings of both beams and truss girders in the system. A discrete set of values has been selected for each variable that makes up the search space from which all solutions to the problem exist. A GA is a search algorithm, incorporated to quickly explore a wide range of answers and focus on better areas of the search space to find improved solutions.

INTRODUCTION

A recurrent problem that structural engineers face in building design is lacking a method for finding the optimal geometry of floor framing members to minimize cost. The intent of this paper is to introduce a method of obtaining the most cost efficient floor system in a large commercial or industrial building through optimizing the geometry of the framing members. The design parameters of the floor area include the number of truss girders, height of truss girders, and number of beams supported by the trusses for a uniformly distributed load.

The *number* of trusses spanning across the floor area reflects the *spacing* of the trusses. The number of *beams* determines the number of *panels* in the trusses, where beams are placed at panel points. While maintaining the topology of the truss girders, panels are added and removed, and the truss height is adjusted.

Evolver, a genetic algorithm optimization software program, was chosen for the analysis since it is easy to operate with a brief background of GA and can quickly find improved solutions to the problem. Evolver is used in this study to find the optimal geometry of the framing members that corresponds to the smallest total cost of the steel frame.

The proposed method provides engineers a quick and reliable cost estimate for optimized framing parameters where expenses for material, framing connections, labor, shipping, and overhead and profit are a function of the weight of the steel members.

CHAPTER 1

LITERATURE REVIEW

This following is a list of recent work in structural optimization that relates to this paper. These studies are given particular attention in that the ideas summarized formed the basis of the proposed method.

Pezeshk and Camp (2001) proposed a GA approach to optimize discrete and continuous steel structures. The benefits of GA optimization were compared to other methods. GA was found to be more robust given that it can improve on solutions through fitness scaling. Additionally, genetic algorithms provide an open format for constraints, permit multiple loading conditions for structures problems, are capable of evaluating random data, and can be applied to a wide range of engineering problems including both discrete and continuous types.

Deb and Gulati (2001) simultaneously optimized the size, topology, and configuration of 2-D and 3-D truss structures through a GA to obtain the minimum total weight while complying with stress and deflection limitations. The design variables considered were continuous and were assigned real-number values. A ground truss-structure was also optimized using discrete values for the member areas, and this solution was compared to the continuous solution. A loaded and fully connected ground structure was first assumed containing all nodes of the truss. Truss members were removed from the initial ground structure in order to search for better solutions. Members that were assigned very small cross-sectional areas were removed, thus changing the topology of the truss structure. The nodes that carried loads or were at support points were termed *basic* nodes, whereas other nodes that could more efficiently distribute loads between members were considered *non-basic* nodes. When basic nodes were eliminated through optimization, a large penalty constant was applied to the objective function which inhibited further optimization of that particular solution. The population size chosen for the GA was dependent upon the number of members used in the ground structure. GA proved to find better solutions to problems that had previously been solved in literature using other optimization techniques.

Hamza et al. (2003) optimized the size, configuration, and topology of large-span N-shaped roof trusses through a modified reactive taboo search (RTS) method. The intent was to find the least weight truss using a discrete set of variables with realistic design choices. These variables were divided into member sizing variables and topology and configuration variables. The search process began with a loaded and entirely connected ground structure from which members with near zero cross-sectional areas were eliminated. Cross-sections were assigned to different member groups of the truss from a list of standard I-, C-, and L-sections. The truss topology options were restricted to N-shaped designs. A fixed roof layout plan was considered from up to five user-defined choices. Stress and slenderness limitations, in addition to purlin spacing and diagonal member angle constraints, were applied and handled through penalty functions. As the objective function was to minimize cost (directly related to weight), a penalty expense was added to infeasible designs in order to bring the cost above the current best answer. It is mentioned that an initial feasible design is important to begin the optimization process in order to effectively apply the penalty function. The researchers compared RTS to genetic algorithms (GA), noting that RTS lacks the diversification capabilities of a GA. This led the authors to develop a diversification mechanism that directs the RTS to search further unexplored areas. The results demonstrated that this modification enhanced the operation of the RTS.

Hayalioglu and Degertekin (2005) presented an optimum design method for the minimum cost of non-linear steel frames with semi-rigid connections and column bases through GA while satisfying stress, deflection, and size constraints. Framing member sizes were chosen from the list of sections provided in the American Institute of Steel Construction design manual. Members were divided into groups to reduce the problem size, and the frame topology remained constant. The final cost of frames produced through Load and Resistance Factor Design (LRFD) and Allowable Stress Design (ASD) were compared to each other, as well as modeling of semi-rigid versus rigid connections and columns bases. The details of rotational stiffness in connections were examined. The researchers noted that generalizations of either fully pinned or fully fixed connections could lead to inaccurate assumptions of frame stability. When setting up the GA, a high crossover rate of 0.95 and a low mutation rate of 0.002 or 0.003 were found sufficient for the problem population sizes of 42 and 56. A tolerance value between 0.002 and 0.008 was used, and through trial and error, a generation size of 500 was found to be an appropriate stopping point for the optimization process. Results proved that the total cost of frames was lower with connections that were less flexible in comparison with more flexible connections. Also, frames with stiffer connections had smaller costs than frames with completely rigid connections. In general, LRFD yielded less expensive designs in comparison to the ASD method, and the total cost of frames could be minimized by adjusting the stiffness of the connections.

Klansek and Kravanja (2006) examined the costs related to composite floor systems. The proposed cost functions associated with estimating the direct manufacturing of the floor systems were explained, including expenses for material, power consumption, and labor. Preparation and installation times for material and the associated expenses were considered in detail. Nonlinear programming (NLP) was used to optimize the costs of composite I-beams, composite trusses with hot rolled channels, and composite trusses with cold formed hollow sections. Comparisons were made between the costs of the three floor systems. The purpose was to determine which floor system governed for certain spans with consideration to different material costs, labor costs, and loading conditions. The conclusions made are as follows: (1) Composite I-beams were found to be the most cost efficient for the majority of spans; (2) Composite trusses made with hot rolled channels governed for cases where higher hourly labor costs were incurred; and (3) Composite trusses made with cold formed hollow sections were not as cost efficient as the other two framing systems and only controlled for a select few conditions.

Optimization of the weight or cost in the design of steel structures is a current topic in engineering research. GA was used to optimize the steel floor system cost in this study for the reasons outlined by Pezeshk and Camp. Also, several other papers referenced in the subject of structural optimization included GA approaches due to its benefits. While much research has been done in truss optimization with consideration to only one problem of size, topology, or configuration, all three are examined for the trusses in this study as in the work of Deb and Gulati and Hamza et al. Since the LRFD method typically produces smaller cost structures than ASD, LRFD has been chosen as the design method for the analysis. Also, unlike many structural optimization methods that give consideration to only material cost in the objective function, this method includes expenses for material, member connections, equipment, labor, shipping, and overhead and profit, similar to the approach suggested by Klansek and Kravanja.

CHAPTER 2

PROBLEM DEFINITION

Overview of Design Method

Three variables for each of two truss topologies are fluctuated in search of the minimum cost of a steel floor system for a fixed area: the number of beams n_b, the number of truss girders n_t, and the height of the truss girders h_t. Figure 1 illustrates the floor framing system discussed. The beams and truss girders are equally spaced over the floor area with the same size beams and trusses used throughout. Both beams and trusses are designed for the interior spans even though the end spans carry half of the load. This is done for simplicity of design and construction. The beams separate at each truss, connecting onto the sides of the truss girders so that the top flanges of the beams are at the same level as the top chords of the trusses. A composite metal deck spans across the floor area. The ribs of the metal deck run perpendicular to the beams and are treated as a continuous linear load across the beams. The beams and truss girders are both considered simply supported, where the truss girders are the primary framing members. To maintain symmetry in the geometry of the Pratt and Howe trusses, an odd number of beams rest across them, with a vertical web member at the location of each beam.

Methodology

Components of an optimization problem include an objective function, unknowns, and constraints. This study is modeled as a joint engineering and expense problem. The economical model is written in terms of the dependent and independent variables and correlated expense coefficients.

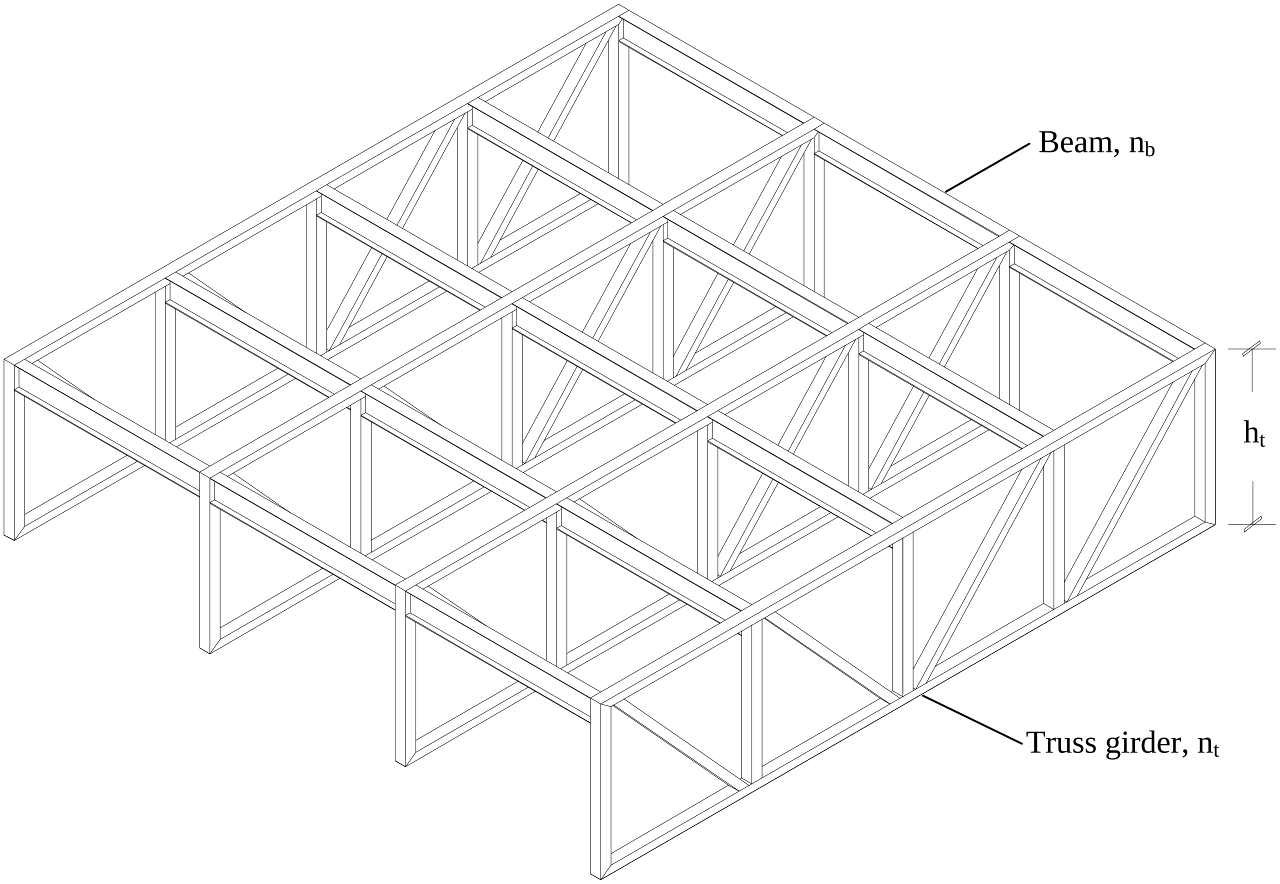


Figure 1: Steel framing system.

The problem variables are independent of one another with the exception of constraint limitations such as the connection angle of the truss branches, which is affected by the truss height and beam spacing. The cross-sectional areas of the beams and truss members vary depending on the values of the three primary design variables. The total cost of the floor framing can be correlated to the weight of structural steel used in the system. Each cost coefficient is directly related to the weight of steel used for the beams and the trusses.

Objective Function

The objective function of this study, cost, is to be minimized. Total cost includes expenses for material, framing connections, labor, shipping, and overhead and profit. Cost is associated with the weight of the framing members. A cost coefficient for each beam is multiplied by the weight of each beam, and a separate cost coefficient for trusses is multiplied by the weight of each truss as shown in the following equation.

$$\min C = \Sigma\, c_b W_b + \Sigma\, c_t W_t \qquad (2.1)$$

Unknowns

The primary design variables for this optimization problem include the number of beams, the number of truss girders, and the height of the truss girders. The spacing of the beams controls the panel layout of the trusses since it is preferred to place beams at panel points (McCormac 87). The rectangular dimensions x_1 and x_2 of the floor area are designated by the program user, and the loading applied to the beam may also be varied. The lighter of the two truss topologies fluctuates depending on the values of each of these parameters.

The ratio of panel length to truss height resembles the aspect ratio of stiffener spacing to the distance between flanges in plate girders. For efficient stress distribution, the panel length in the trusses decreases as the truss height increases, just as the spacing between stiffeners decreases as the distance between flanges increases in plate girder design. The behavior of Pratt trusses in particular is similar to that of plate girders in that the tension fields exist in the same locations. Tension in plate girders can be observed when the web buckles under diagonal compression, exposing the tension-field (Salmon 640).

Constraints

The objective function is subject to a set of design and behavioral constraints on the optimization problem, which have been deliberately chosen by limits set on the basis of what is practical in the engineering environment. These constraints define the physical boundaries of the variables and are written in the form of equality or inequality functions. The types of steel used throughout the layout and properties of the steel are material limitations. ASTM A992, high-strength, low-alloy, wide flange beams are used with a minimum yield stress F_y of 50 ksi and tensile strength F_u of 65 ksi. ASTM A500, cold-formed, welded and seamless, carbon steel structural tubing, grade B, HSS truss members are used with $\mathrm{F_y}$ = 46 ksi and $\mathrm{F_u}$ = 58 ksi (AISC Table 2-1).

Design constraints include the boundaries of the design variables. The minimum number of beams is *three*, which produces the minimum number of panels in a Pratt or Howe truss. The maximum number of beams has been chosen through trial and error as *19*. At first this was chosen as *65*, a high arbitrary number, so that the spacing of the beams would be very tight and the search space would be large. After running several trial optimizations with Evolver, the best solutions tended to have less than 15 beams. *Nineteen* was chosen as the maximum number of beams in order to increase the probability of better answers. Evolver works more efficiently if the search space can be reduced. Additionally, only odd numbers of beams are chosen to maintain symmetry in the panels of the Pratt and Howe trusses.

Similar to the beams, the minimum number of trusses is chosen as the low value of *three*. The maximum number of trusses has also been chosen as *19* for the same reasons as for the beams. Also, only integer numbers of trusses must be chosen.

The minimum truss height has arbitrarily been chosen as the small value of *3 ft*. A shallower truss than this did not seem reasonable since deeper trusses usually require less material. A maximum truss height of 15 ft. has been chosen to limit transportation difficulties under bridges on roadways. The truss height is chosen in increments of whole feet so that values can easily be chosen from AISC tables. Also, for ease of welding, the branch member orientations are maintained greater than 30 degrees (AISC Spec. for Steel HSS 9.4). Therefore, the ratio of the height of the trusses to the spacing of the beams must be greater than $3^{½}/3$ and less than $3^{½}$.

Behavioral constraints include stress, strain, and deflection limitations. The total and live load deflection limits need to be satisfied for both beams and trusses. The calculated plastic section modulus must be less than the plastic section modulus of the shape chosen. Additionally, slenderness and design strength limitations must be satisfied for both compression and tension members. The following is a list of the design and behavioral constraints considered in this study.

Design Constraints

- $n_b = \text{odd}(n_b)$ (2.2)
- $\frac{\sqrt{3}}{3} < \frac{h_t}{s_b} < \sqrt{3}$ (2.3)
- $3 \leq n_b \leq 19$ (2.4)
- $3 \leq n_t \leq 19$ (2.5)
- $3' \leq h_t \leq 15'$ (2.6)

Behavioral Constraints

Flexure

- $\Delta_{TL} < L/240$ (2.7)
- $\Delta_{LL} < L/360$ (2.8)
- $Z_{x,c} < Z_x$ (2.9)

Tension

- $L/r_y \leq 300$ (2.10)
- $P_c < \phi_t P_n$ (2.11)

Compression

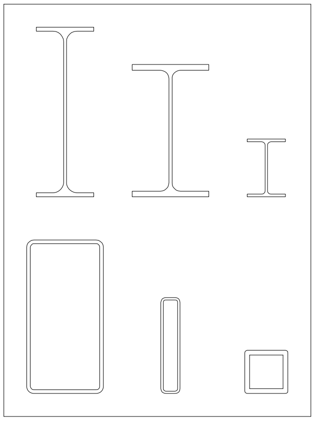
- $K\ L/r_y \leq 200$ (2.12)
- $P_c < \phi_c P_n$ (2.13)

Parametric Optimization Problems

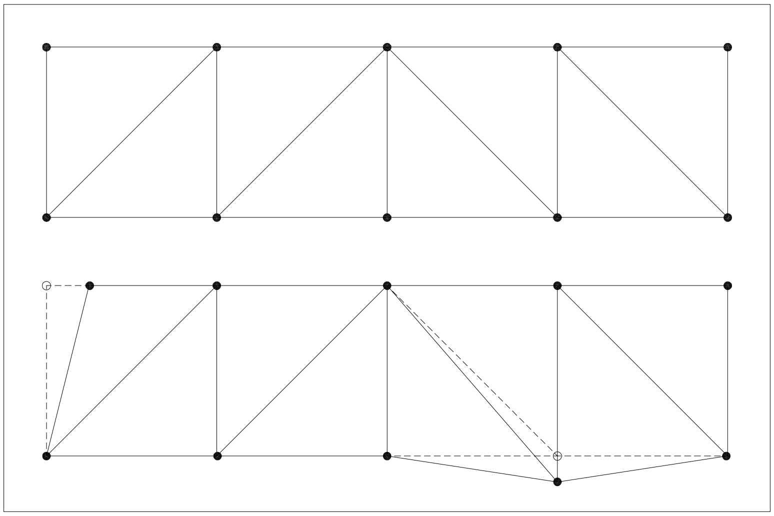
Parametric optimization is defined in this paper as the combination of configuration, size, and topology optimization of a truss girder, the size optimization of a beam, and the optimization of the spacings of both beams and truss girders in the system.

Figure 2 shows examples of the three basic truss optimization problems: size, configuration, and topology. Size optimization is where the member cross-sectional areas of a truss are optimized. This is the most commonly solved problem in research. Configuration optimization is where the truss joint locations are optimized. In topology optimization, the connectivity of the joints is optimized (Hamza et al.).

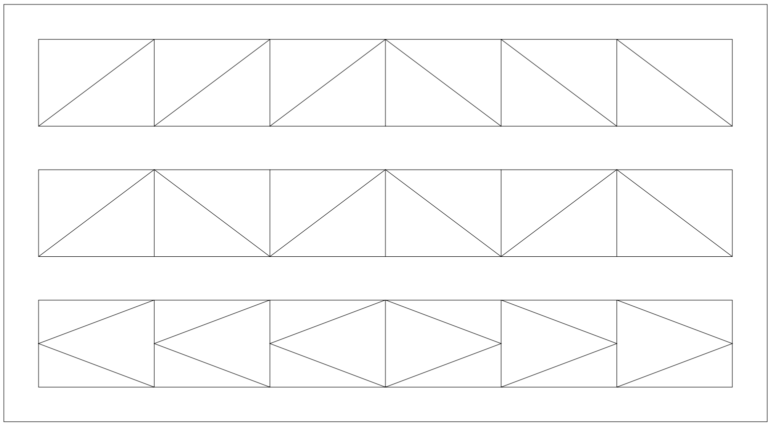
Two topologies are considered for the truss girder in order to limit the complexity of the problem. A medium-rise Pratt truss and Howe (inverted Pratt) truss have been chosen for the floor system given that they can efficiently span 90 to 100 ft. (McCormac 85). Figure 3 shows examples of two-, four-, and six-panel Pratt and Howe trusses. The truss girders are always comprised of an even number of panels in order to maintain symmetry, and the diagonals of the truss never change orientation. The floor system is symmetrical because the loading considered



(a) Size optimization.

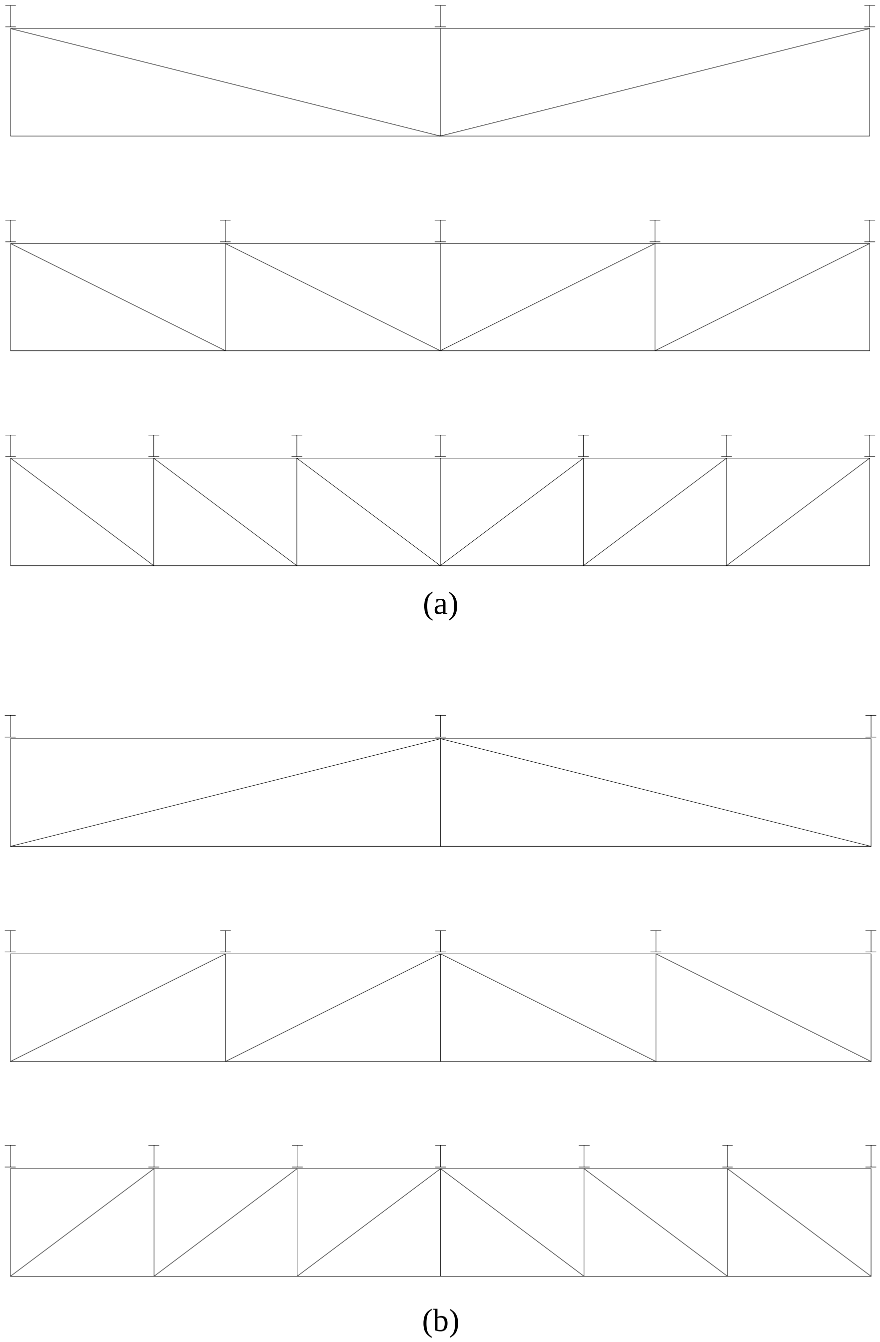


(b) Configuration optimization.



(c) Topology optimization.

Figure 2: Basic truss optimization problems.



(a)

(b)

Figure 3: Two-, four-, and six-panel (a) Pratt truss and (b) Howe truss.

is uniform across the entire floor area. This loading will produce symmetrical shear and bending moment diagrams in both the beams and truss girders.

In addition, hollow structural sections (HSS) have been chosen for all of the members of the truss girder because of their high strength-to-weight ratio. Cross-sectional areas are restricted to those rectangular (and square) shapes listed in the Third Edition of the AISC Manual of Steel Construction.

Search Space

The *search space* consists of any feasible solution to the optimization problem. The global optimal solution to a problem is the “best” answer within the entire search space. This is the solution that the problem solver is attempting to find. A local optimum is only the best solution in a particular region of a search space. It is easy to become trapped in a local optimum, thinking that this is the most optimal solution. Therefore, several trials of an optimization algorithm should be examined in the search for better answers. Figure 4 is an example of a search space. The vertical axis represents the objective function, cost, and the horizontal axis represents the set of all variables (n_b, n_t, h_t) that structure each solution.

Deterministic and Non-deterministic Problems

A deterministic problem generates a certain solution based on an initial condition. The element of randomness is removed, & selection is established by a set of rules. A deterministic attribute of the problem in this particular study is how the cross-sectional areas of the members are chosen; the initial condition is that a member is automatically assigned the smallest cross-sectional area for which all design and behavioral constraints are satisfied.

A non-deterministic problem has no certain solution because it progresses through a method of randomness and probability. The number of beams, number of trusses, and truss height are all parameters that can have several solutions. These values are altered randomly while combining traits of better solutions in order to expand the search as well as to fine-tune the search in more favorable areas (Reed 158).

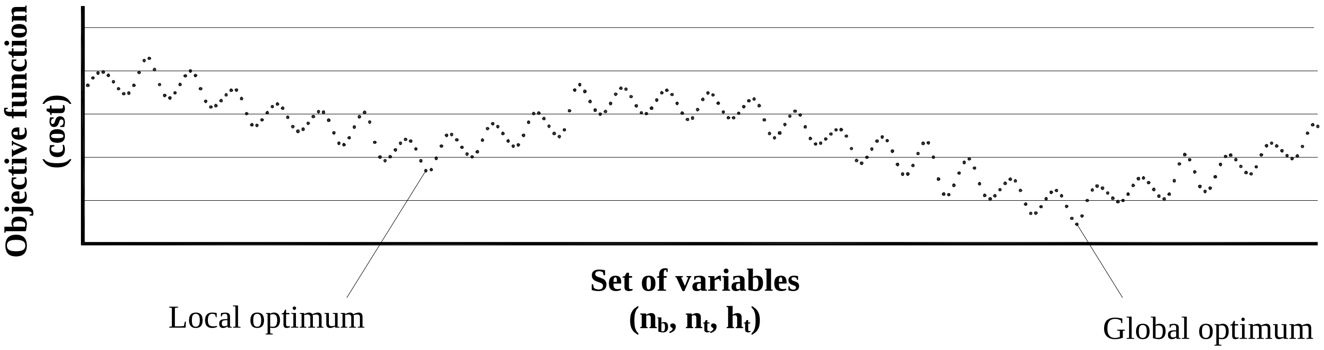


Figure 4: Example of a search space.

Continuous and Discrete Problems

Both deterministic and non-deterministic problems can be either continuous or discrete. A problem that is structured using a continuous method could have an infinite number of solutions since variables may be fluctuated in infinitesimal increments. When a discrete method is implemented, a distinct set of values is chosen for each variable.

A discrete approach is employed in many aspects of this problem. Member shapes and sizes are chosen from the list of common steel sections provided in the AISC manual. The height of the truss is varied in increments of whole feet and the number of trusses and beams is varied in whole integer numbers (with the beams adjusted in odd numbers). Two truss topologies are analyzed, the Pratt & Howe designs. A discrete number of other common topologies may have also been considered, such as the Warren truss and K-truss (Nazareth 93, 94).

A continuous approach could have been taken in several areas of this problem. If it was applied to size optimization, cross-sectional areas not included in the AISC manual may have been used. Web and flange thicknesses, widths, and depths other than the more commonly seen dimensions might have been considered. The cross-sectional shape of members could have been distorted to limitless possibilities, or the cross-section could vary throughout a member to reflect the non-uniform stresses acting within it. The spacing of the beams and trusses and height of the trusses would have been varied in decimal inches. The topology of a highly connected truss with several joints could have been modified as a continuous function, where members would be removed or added to find the optimal solution. Another continuous method for topology optimization is where a supported dense mass of material representing a truss with a load applied to it is gradually “dissolved” throughout its volume to produce several different solutions. Eventually an efficient load path would be found through the truss to its supports. This would also be a form of size and configuration optimization. A drawback to the continuous approach is that the solutions produced, including the optimal solution, are not usually practical or easily constructible and therefore not cost effective. Modifications generally need to be made to the final solution to create a realistic design when a continuous method is employed.

Program Setup

The Excel program developed for this study is composed of four individual tables. These consist of the *main design and analysis table* where the design parameters are chosen and the majority of calculations are done, the *beam selection table* which is a list of section sizes available for the beam members, and the *compression member selection table* and *tension member selection table* where a list of sections are provided for each truss member. A diagram of the program setup is shown in Figure 5. The Excel program automatically performs a structural analysis for any floor framing geometry, and a total cost is immediately given for the floor system.

Loading

The dead load applied to the floor system includes a 115 pcf lightweight concrete slab with a chosen 4-inch minimum thickness poured into a steel deck and an additional 20 psf uniform dead load for mechanical duct work. As the focus of this study is placed on the cost of the steel members of the framing system, the concrete slab is not optimized. Camp et al. (1999) developed a method that utilized a GA to optimize the minimum cost of shear connectors for composite action between steel beams and concrete slabs. While the concrete slab is assumed to be part of a composite floor system, it should be pointed out that for simplicity, it is designed as a stand-alone slab without steel deck. This is because the allowable span of a composite deck depends on the type of steel deck utilized and does not increase linearly with the thickness of the slab. An example of a composite floor deck is illustrated in Figure 6.

The maximum length the concrete slab can span between beams while supporting its self weight in addition to the service loads determines its thickness. This thickness is adjusted in increments of whole inches. While the amount of concrete is not calculated into the cost of the framing system, its weight increases the total load the steel members must support. The maximum amount of reinforcement permitted by the ACI design code establishes the maximum span of the slab. These calculations are given in Appendix A.

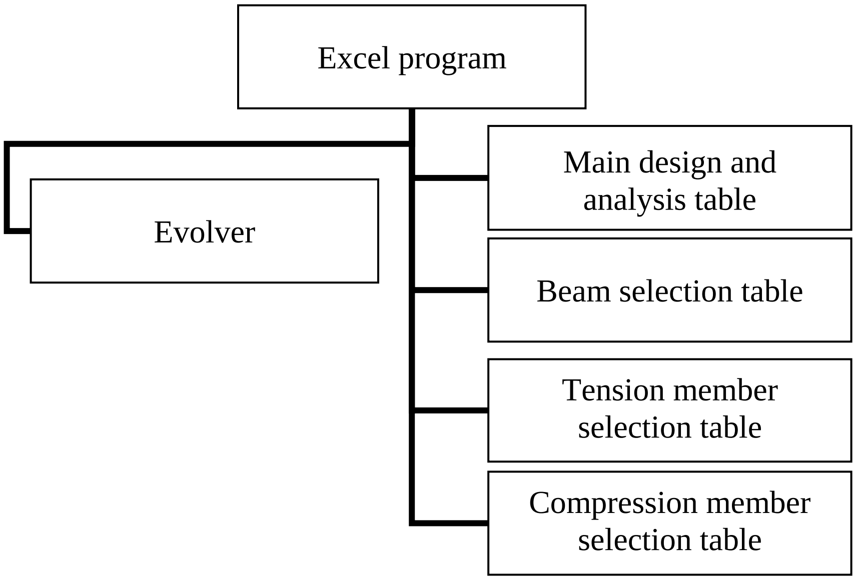


Figure 5: Design program setup.

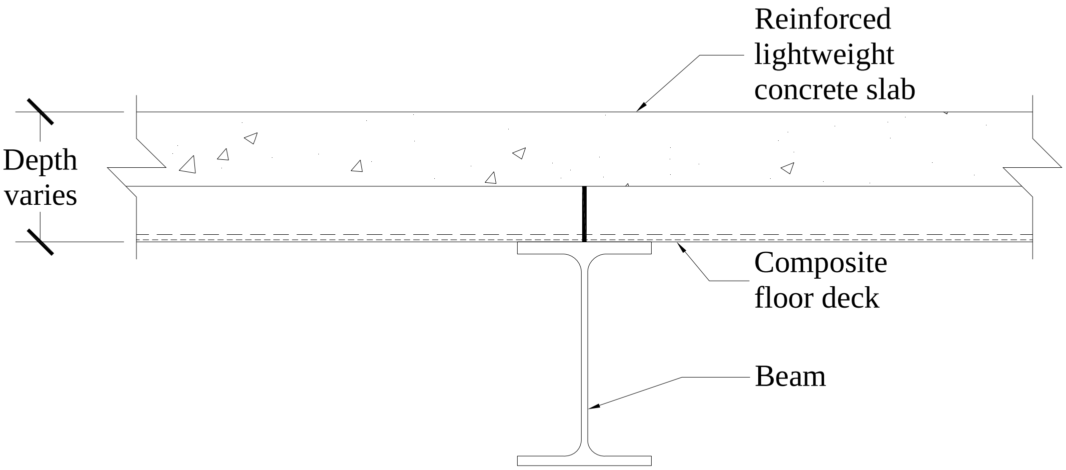


Figure 6: Composite floor deck.

For a concrete compressive strength f'_c of 3 ksi, a factor β_1 of .85 is taken as a design assumption (ACI 10.2.7.3). For Grade 60 (F_y = 60 ksi) reinforcement, the compression-controlled strain limit ε_u is equal to 0.002 (ACI 10.3.3). The net tensile strength ε_t in the extreme tension steel at nominal strength is taken as F_y/E, but no less than 0.004 (ACI 10.3.5). The maximum reinforcement ratio is determined by

$$\rho_{max} = .85\beta_1 \frac{f'_c}{F_y} \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_t} \tag{2.14}$$

(Nilson 81). The strength reduction factor for bending ϕ_b is taken as 0.9 (ACI 9.3.2.1). Using a ¾ inch concrete cover, slab thickness *h*, and assumed #5 bar diameter, an effective depth *d* is found for a section of slab of width b = 1 ft (ACI 7.7.1(c)). The flexural resistance factor is given by

$$R = \rho_{max} F_y \left(1 - .59 \frac{\rho_{max} F_y}{f'_c} \right) \tag{2.15}$$

which is used to obtain the moment design strength

$$\phi_b M_n = \phi_b\ R\ b\ d^2 \tag{2.16}$$

(Nilson 83, 84). The maximum span is taken as

$$l = \min\left(\sqrt{\frac{8\phi M_n}{w}}, 28h \right) \tag{2.17}$$

(ACI Table 9.5(a)) . For any thickness chosen, 28 h governs. Therefore, the maximum span of a 4-inch slab is 9'-4", and for every additional 28-inch span length, one inch is added to the slab thickness and incorporated into the total load. The variance in slab thickness for different loading conditions is shown in Figures 7 – 9. Because there is only a slight increase in slab thickness for each loading condition, it is concluded that the spacing and span of the beams is highly dependent on the weight of the slab.

Although the program may either use floor or roof trusses in the analysis, the focus of this paper is on floor trusses. The uniformly distributed live load that is used in calculations is taken from the 2004 Florida Buildings Code (FBC) as 100 psf for an office lobby (FBC Table 1607.1).

If these are assumed to be flat roof trusses, a typical roof live load L_r, may be found as follows, where R_1 and R_2 are reduction factors:

$$L_r = 20 \text{ psf } R_1 R_2 \tag{2.18}$$

$$R_1 = 1 \tag{2.19}$$

$$R_2 = 1 \tag{2.20}$$

(FBC Eq. 16-24, Eq. 16-25, Eq. 16-25).

Although the load and resistance factor design method is implemented, load factors are not used in conjunction with the service loads. Strength and serviceability limit states are not exceeded in design. Strength limit states involve the maximum load carrying capacities of members, whereas serviceability limit states influence the performance of members under typical loading conditions (AISC Sect. A5.2).

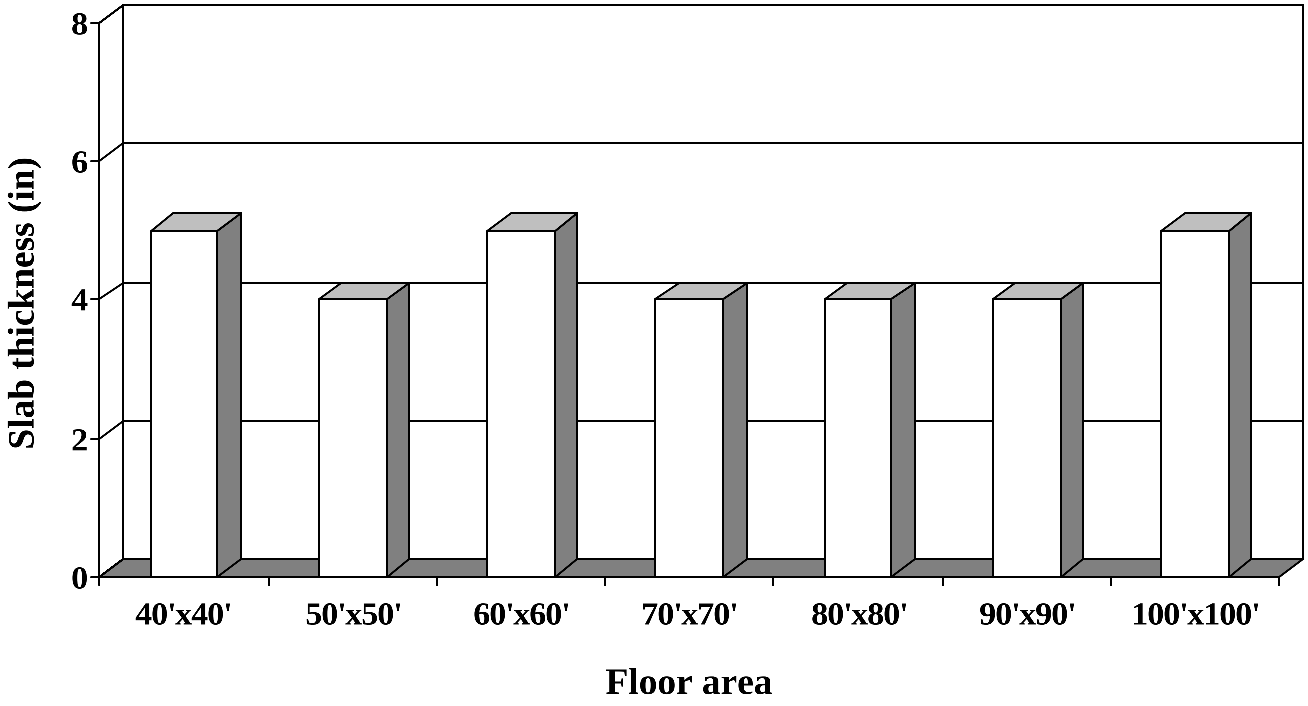


Figure 7: Slab thickness for optimized floor framing cost, 60 psf load.

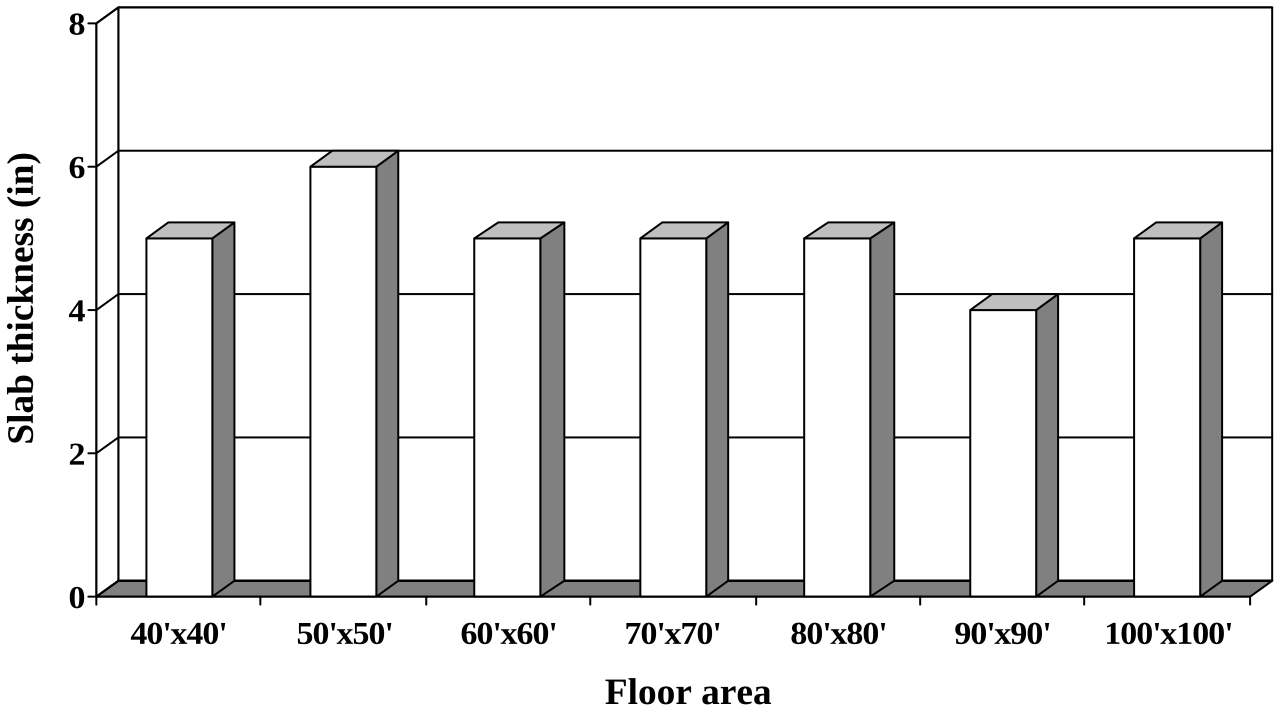


Figure 8: Slab thickness for optimized floor framing cost, 120 psf load.

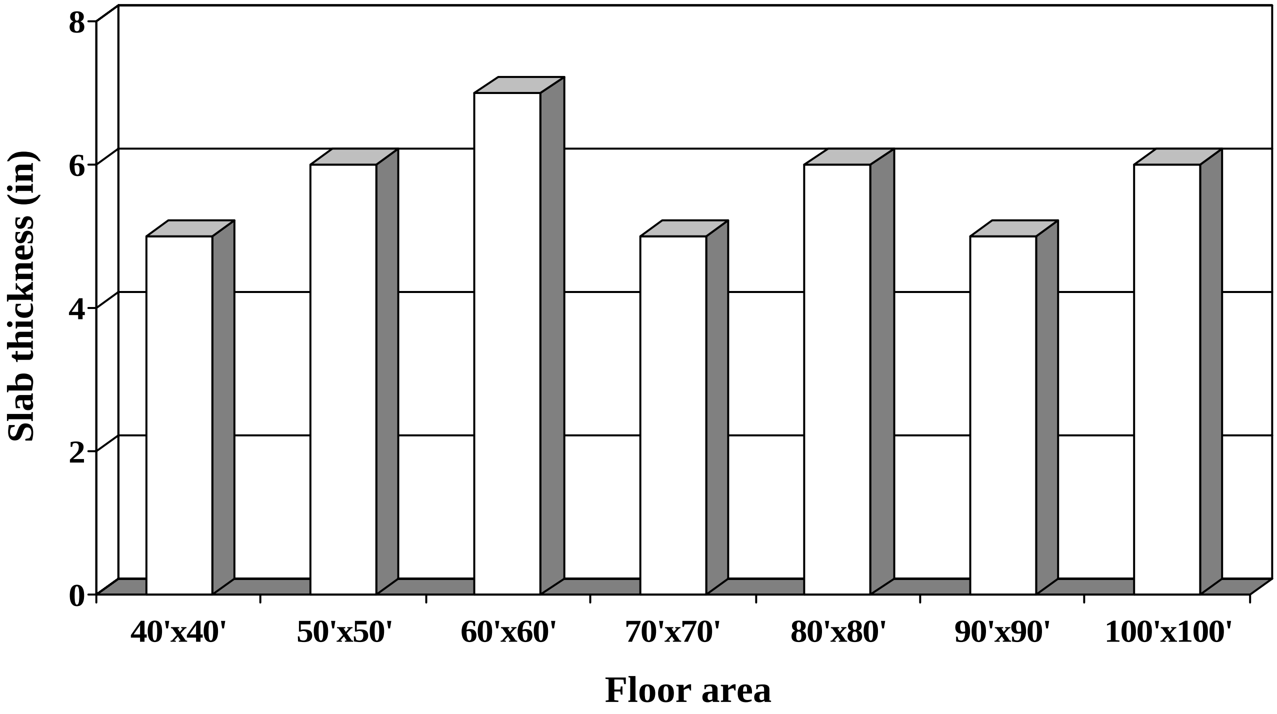


Figure 9: Slab thickness for optimized floor framing cost, 240 psf load.

CHAPTER 3

FRAMING MEMBER DESIGN

The calculation procedure for the Excel spreadsheet created is outlined in Appendix B as a MathCad file. All equations listed in Appendix B must be included in the spreadsheet for the program to correctly perform the structural analysis for any floor framing geometry, although the format of the Excel program is at the discretion of the designer. All equations are inputted into the *main design and analysis table* with the exception of the equations listed under the sections: *beam selection table*, *tension member selection table*, and *compression member selection table.* The following is a detailed description of these calculations.

Wide Flange Sections

The beams used in the flat roof system are W-shaped sections, supported by each truss. The flanges of these sections have parallel inner and outer surfaces (AISC 1-4). Consideration has been given to only those shapes listed in the AISC manual.

Flexural Design

Width-thickness ratio. The beam sections are assumed to be compact; the width-thickness ratio for both the web and flanges do not exceed the limiting slenderness parameter λ_p for each compression element. This is assumed because it is the most common condition, and therefore the limit state of yielding is used for calculations. Had noncompactness been assumed, a penalty factor would have been employed that would make the section heavier than what is necessary. Since minimal weight is desired, the author has chosen the assumption of compactness for the beams.

Lateral-torsional buckling. For bending about the major axis, the design flexural strength, determined from the limit state of lateral-torsional buckling, is ϕM_n. The nominal strength is dependent on the limiting laterally unbraced length, where braced points prevent lateral-torsional buckling of the top and bottom flanges. These braced points occur at each beam-to-truss connection. The uniformly distributed concrete slab provides lateral and torsional bracing to the beams continuously along the top flanges as well as to the top chords of the trusses (AISC 5-6, Sect. C3.4).

Yielding. Because the beams are considered compact and braced, the flexural limit state of yielding controls the flexural design strength (AISC Ex. 5.1). With application to the limit state of yielding, the nominal flexural strength M_n is determined from the moment due to service loads M and the resistance factor for flexure ϕ_b.

$$\phi_b = 0.90 \tag{3.1}$$

$$M_n = M/\phi_b \tag{3.2}$$

This is equal to the plastic bending moment M_p.

$$M_n = M_p \tag{3.3}$$

The required plastic section modulus Z_x is found by M_p and the minimum yield stress F_y.

$$F_y = 50 \text{ ksi} \tag{3.4}$$

$$Z_x = M_n/F_y \tag{3.5}$$

(AISC F1.1, Table 2-1). The required moment of inertia I_x is established as the greater produced between total load and live load deflections at midspan. The required area A of the beam is chosen from the Excel *beam selection table* and multiplied by 3.4 psi/ft to convert to lb/ft. This is determined from the unit weight of steel, 490 pcf, times one square foot of steel.

Shear design strength. Because the shear design strength does not usually govern for large spans in comparison to bending and deflection, this check is therefore omitted.

Deflection. The required moment of inertia I_x is established as the greater produced between total load and live load deflections at midspan. The required area A of the beam is chosen from the Excel *beam selection table* and multiplied by 3.4 psi/ft to convert to lb/ft.

Beam Selection Table

A, I_x, and Z_x are inputted into the Excel *beam selection table* for each section listed in AISC Table 1-1. For each section, if its I_x is greater than or equal to the required I_x, then its area is selected in a separate column. Otherwise, a default area of 9,999 in^2 is selected. Also for each section, if its Z_x is greater than or equal to the required Z_x, then its area is selected in that same column. Otherwise, the default area of 9,999 in^2 is selected. The maximum area of the two cases is displayed for each section. The smallest area of these represents the least-weight member that satisfies the criteria. The self-weight of this member is then added to the dead load and a new required I_x and Z_x is determined. The process is repeated and a new section is found if the previous section was insufficient for the addition of its self-weight. Typically, the serviceability limit state of deflection governs for longer spans. Each time a cross-sectional area is selected for the beams in the *beam selection table*, this chosen area is linked to the *main design and analysis table* to include with the total weight of the floor system.

Hollow Structural Sections

HSS truss members have been chosen for the analysis because of their high strength-to-weight ratio. Lena Singer, writer for *Modern Steel Construction Magazine*, comments that HSS members in design can save on material costs, shipping, handling, and erection expenses, and with less member surface area, also fireproofing/painting costs. HSS also performs well under compression. Although HSS members cost approximately 10 cents more per pound than wide flange members, HSS can weigh nearly 40-percent less than W-sections with comparable strengths (Montgomery, Williams, Singer). Welded joint connections are usually more costly and complicated than bolted connections, but these costs are curtailed through minimizing the size of the section to be welded. Since HSS are not typically used as single-beam framing members, a different shaped section is used for the beams.

The type of HSS chosen for design has a rectangular (or square), prismatic shape with rounded corners. A rectangular section has two webs and two flanges. The wall thickness and cross-sectional shape are constant throughout each member.

Only consideration has been given to those shapes listed in the AISC manual, which are comprised of “unstiffened non-composite HSS in non-fatigue applications” (AISC Spec. for Steel HSS 1.1). All members of the truss have a width-thickness ratio less than the thin-walled section classification limit.

Basic assumptions about the strength of members include (1) uniform compressive stress-strain properties throughout a section of a member; (2) no residual stresses from plastic deformation during fabrication of members; (3) perfectly straight and prismatic members; and (4) non-eccentrically applied loads (Salmon 278).

Truss Girder Description

The truss member forces are found through the *method of joints*. Only half of the truss members are solved since the truss is symmetrical. Although a single member is used for the entire top compression chord and a single member is used for the entire bottom tension chord, each chord segment between beams is solved as a separate member. The largest force found in each chord segment is used to size the entire chord member. This chord segment is located closest to the center vertical member. The web members may differ in cross-sectional area. The interior beams each exert the same point load across the truss girders. The end beams, while weighing the same as the interior beams, carry half of the service load applied to them, and therefore a smaller point load is used at the ends of the truss girders.

With an odd number of vertical members varying between 3 and 19, a pattern for automatically solving the forces in the truss members has been established. Starting from the ends of the truss and moving toward the center, the truss member forces are solved.

In a Howe truss, the center vertical member is a zero-force member, tending toward tension. This is sized according to the slenderness ratio for tension, where

$$l/r_y \leq 300. \tag{3.6}$$

The other zero-force members are located at the top chord segments furthest to the truss ends, tending toward compression. This is sized according to the slenderness ratio for compression, where

$$K\ l/r_y \leq 200 \tag{3.7}$$

(AISC Spec. for Steel HSS 2.3). In a Howe truss, the interior vertical web members are in tension while the diagonal members are in compression. The opposite occurs in a Pratt truss. In either situation, the end vertical members are in compression.

Each panel of the truss girder consists of four members. For three beams, only one panel is designed. For five beams, two panels are designed. For more than three beams, a counting factor is used in the *main design and analysis table* to include each additional panel’s set of members that ranges from 1 additional panel (for 7 total beams) to 30 additional panels (65 beams).

$$\text{Number of additional panels} = (\text{Total number of beams} - 1)/2 - 2 \tag{3.8}$$

Tension Member Selection Table

Effective area. The effective area A_e for a welded connection, continuous around the member is equal to the gross area A_g times the shear lag coefficient U.

$$U = 1.0 \tag{3.9}$$

$$A_e = A_g U \tag{3.10}$$

(AISC Spec. for Steel HSS 2.1). The continuous weld around the member can be perceived as two longitudinal and two transverse welds.

Design tensile strength. For welded end connections, the design strength for tension members is calculated based on the limit-state of yielding on the gross section. The tensile design strength must equal or exceed the factored axial tension. Due to variations in material properties and dimensional tolerances, and since the actual strength could be less than the theoretical strength, the resistance factor for tension ϕ_t is employed.

The radius of gyration r_y and the gross area A_g are inputted into the Excel *tension member selection table* for each of the sections listed in AISC Table 3-4. For yielding, the tension member design strength is,

$$\phi_t P_n = \phi_t F_y A_g, \tag{3.11}$$

where

$$\phi_t = 0.9 \tag{3.12}$$

(AISC Spec. for Steel HSS Eq. 3.1-1). After slenderness limitations are verified, each section's design strength is checked to ensure that it equals or exceeds the tension force applied to it. If either criterion is not satisfied, the section is assigned an area of 9,999 in^2 in a separate row in the *tension member selection table*. Otherwise, the section's gross area is displayed in the row. The smallest area displayed for each of the cross-sections corresponds to the least-weight section that satisfies the criteria. Every cross-sectional area selected for the tension members in the *tension member selection table* is linked to the *main design and analysis table* to include with the total weight of the floor system.

Compression Member Selection Table

Flexural and local buckling. Either flexural buckling or local buckling can govern the strength limit state. Flexural buckling (Euler elastic buckling) controls for long compression members, and local buckling (instability) controls for short compression members. The limiting slenderness ratio, minimum yield stress, residual stresses, and initial out-of-straightness are provisions controlling the flexural buckling strength (AISC Spec. for Steel HSS 4.2). A combination of both buckling states occurs in compression members of intermediate lengths (inelastic buckling).

$$\text{Compact:} \quad \lambda \leq \lambda_p \tag{3.13}$$

$$\text{Noncompact:} \quad \lambda_p < \lambda \leq \lambda_r \tag{3.14}$$

$$\text{Slender element:} \quad \lambda_r < \lambda \tag{3.15}$$

λ: slenderness parameter

λ_p: limiting slenderness parameter for compact element

λ_r: limiting slenderness parameter for noncompact element

A compression member can have a compact, non-compact, or slender-element section. A section is *compact* when the plastic moment may be attained without the occurrence of local buckling. Classification of *non-compactness* occurs when the plastic moment is attained with local buckling following initial yielding. When the plastic moment is attained with initial yielding following local buckling, the section is *slender-element*. As in plate design, the square of the width-thickness ratio governs local buckling of rectangular HSS (AISC 16.2-30).

Limiting slenderness ratio. Because joint translation is prevented and the ends of the members are restrained from movement, the truss is considered braced, and the effective length factor *K* is less than one (Salmon 309). The following slenderness and effective length limitations apply to HSS trusses with branch members fully welded to chord members:

$$\text{Branch members:} \quad K = 0.75 \tag{3.16}$$

$$\text{Chord members:} \quad K = 0.9 \tag{3.17}$$

(AISC Spec. for Steel HSS 3.1). These values have been used in calculations. It is permitted by AISC to assume that the welded HSS truss members are pinned-pinned connected. This is a more conservative assumption.

Design compressive strength. The compressive design strength $\phi_c P_n$ for compression members is given by

$$\phi_c P_n = \phi_c F_{cr} A_g, \tag{3.18}$$

where the resistance factor for compression is $\phi_c = 0.85$. The critical stress F_{cr} is determined as follows:

For $\lambda_c \sqrt{Q} \leq 1.5$,

$$F_{cr} = Q\,0.658^{Q\lambda_c^{\,2}} F_y \qquad (3.19)$$

For $\lambda_c\sqrt{Q} > 1.5$,

$$F_{cr} = \frac{0.877}{\lambda_c^{\,2}} F_y \qquad (3.20)$$

Where

$$\lambda_c = \frac{K\,l}{r \cdot \pi}\sqrt{\frac{F_y}{E}} \qquad (3.21)$$

(AISC Spec. for Steel HSS 4.2).

Although AISC Table 4-6 does not include every section listed in AISC Table 1-11, the Excel *compression member selection table* includes the full list in Table 1-1. Dr. Serge Zoruba, Senior Engineer at AISC, commented that only typical column sizes are listed in the AISC Compression Member Selection Tables due to space limitations in the Manual. Zoruba also mentioned that three significant digits were maintained throughout calculations for the design strengths, accounting for deviations between answers that others may obtain. While most of the design strengths calculated in the Excel *compression member selection table* are similar or identical to the values found in the Manual, larger differences in answers occur more often among large HSS sections with small effective lengths. The *compression member selection table* answers are generally more conservative. The calculated weight per foot for some of the sections differed slightly from the nominal weight per foot listed for each section in the Manual. All weights have been calculated using the areas listed in the Manual multiplied by 3.4 psi/ft. Equations directly from the Manual are used to calculate the values for the *compression member selection table*, and exact values are used throughout calculations. For each section, the thickness t, depth b, radius of gyration r_y, slenderness parameter λ, and gross area A_g are listed. λ is the depth to thickness ratio of the cross-section, where the actual depth is taken as the total section width minus three times the thickness (AISC Sect. B5.1).

If the ratio of the effective area A_e to the gross area A_g is greater than one, the full reduction factor Q is taken as one. Otherwise, Q equals the ratio, A_e/A_g (AISC App. B5.3c). In otherwords,

for $A_e/A_g > 1$,

$$Q = 1 \qquad (3.22)$$

otherwise,

$$Q = A_e/A_g. \tag{3.23}$$

Furthermore,

$$A_e = A_g - 4\,(b - b_e)\,t, \tag{3.24}$$

where the effective depth b_e is determined as follows:

$$f = \phi_c\,F_{cr}. \tag{3.25}$$

For $\lambda \geq 1.40\sqrt{\frac{E}{f}}$,

$$b_e = 1.91t\sqrt{\frac{E}{f}}\left(1 - \frac{0.381}{\frac{b}{t}}\sqrt{\frac{E}{f}}\right) \tag{3.26}$$

otherwise,

$$b_e = b \tag{3.27}$$

(AISC App. B5.3b). In order to determine $A_{e,}$ Q is assumed to equal one when finding F_{cr}. The A_e of each section for each effective length is found, and then Q is found for each A_e.

Similar to the *tension member selection table,* after slenderness limitations are verified, each section's design strength is checked to ensure that it equals or exceeds the compression force applied to it. If either criterion is not satisfied, the section is assigned an area of 9,999 in^2 in a separate row in the *compression member selection table*. Otherwise, the section's gross area is displayed in the row. The smallest area displayed for each of the cross-sections corresponds to the least-weight section that satisfies the criteria. The smallest cross-sectional area selected for each of the compression members in the *compression member selection table* is linked to the *main design and analysis table* to include with the total weight of the floor system.

Deflection

The deflection of a truss is controlled by the displacement of its joints. Using the method of virtual work, a downward unit point load P is applied to the center of the top chord of the truss girder. The virtual internal force in each truss member is then calculated through the method of joints. Next the deflection of each joint is found through

$$\Delta = \frac{nN\,L}{A_g E\,P} \tag{3.28}$$

where *n* is the virtual internal force due to the unit load and *N* is the internal force due to the real loads. The sum of all joint deflections is compared to the allowable deflection of the truss (Hibbeler 305).

Total load and live load deflections are both evaluated. When calculating the live load deflection at each joint, *n* remains the same as for the total deflection, and the live load is used to calculate *N* for each member.

If deflection is not satisfied for a particular solution, the chord members of the truss are enlarged to the next smallest section and deflection is again checked. This is because the chord members carry the largest forces of the truss and contribute the most to deflection. The process is repeated several more times until deflection is satisfied. If deflection is never satisfied after enlarging the size of the chords 20 times, the weight of each truss will have increased from anywhere between 20-50%. At this point, a heavier truss than this most likely will not produce a smaller cost floor system than what Evolver will have found. This solution is then discarded.

CHAPTER 4

CONNECTIONS

Truss branch members are continuously welded around the perimeter to chords, creating both in-plane and out-of-plane end restraint. As larger welds are more costly, it is more efficient to choose thinner branches. The weight of the welding material is considered negligible when adding the total weight of the truss girder. Crossing diagonal members are avoided by reason of additional welding costs. Gapped joints are assumed because they are less expensive to weld and easier to fabricate. The chords are partially restrained from rotation due to welded branch members.

The effective length factors for compression truss members listed in equations 3.16 and 3.17 of this paper have been included with the calculations, although truss branches may be assumed to be pinned as permitted by the steel manual (AISC Spec. for Steel HSS 4.1). All other members are treated as pinned-pinned connected, as well as the trusses. The trusses are assumed to be supported between the columns. A specific connection is not called out for the truss-column connection since the design of the columns is out of the scope of this research. The type of column used would determine the type of connection fabricated.

The connection of beams to trusses can be made with a minimum ¼ inch fillet weld applied in the field to the web of the beam. The thickness of the weld depends on the thickness of the members and loading conditions. These are assumed to be pinned-connected for the analysis. Depending on the applied load, a single vertical weld on one side of the web of the beam is usually sufficient. For this type of connection, coping of members is not necessary, saving in fabrication costs. Neither fin plates nor angles are used to make the connection, which also saves money. A typical truss joint connection and beam-truss connection is shown in Figure 10.

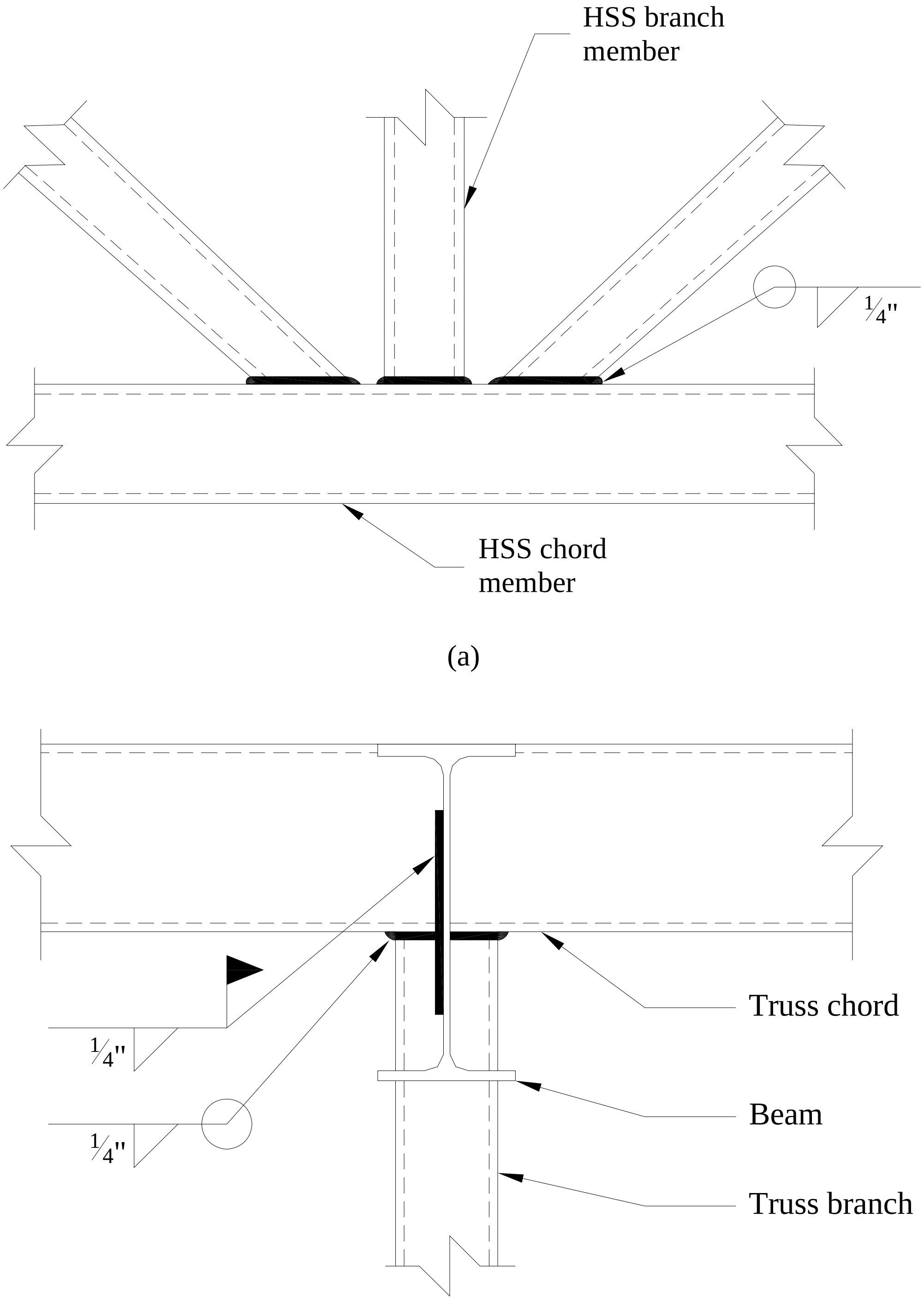


Figure 10: Typical connections, (a) truss joint connection and (b) beam-truss connection.

CHAPTER 5

COST ANALYSIS

Material costs are based on the weight and shape of members. HSS sections tend to cost more than W-sections, and smaller sections tend to cost more than larger sections. Welding material, bolts, plates, and other fasteners are additional items included with the material cost.

The hourly pay of crew members is included in labor charges and is based on average production rates. The crew comprises engineers, drafters, manufacturers, fabricators, installers, welders, structural steel workers, equipment operators, truck drivers, and supervisors. Labor involved consists of milling, handling and loading, delivery to shop, drafting, engineering, shop preparation and fabrication, warehouse rehandling, trucking to the job site, unloading, erecting, field welding and bolting, and crane installation. This does not include overtime and is based on a typical eight-hour workday.

Equipment costs include average rental rates of high quality, late model machinery, in addition to operation expenses such as fuel, oil, and maintenance. Trucks utilized to transport the framing members, welding equipment, cranes, saws, and other tools are some of the items that are included with the cost.

Overhead and profit expenses take into account the calculated markups applied to laborers’ base pay rates including fringe benefits by the installing contractor, engineer, and other involved employers. Overhead consists of the expenses of operating a business, such as rent, utilities, and other finances not associated with the specific labor. The return amount each business receives for the work makes up the profit amount.

RSMeans

The framing system is assumed to be part of a new commercial or industrial building. For this project size, the *Means Heavy Construction Cost Data* book is referenced in order to develop an accurate and current construction cost estimate for the framing of the floor system. RSMeans bases cost on the use of quality materials and workmanship. Material, labor, and equipment costs and an allowance for the installing contractor's overhead and profit are included with the final estimate of the framing system. Expense for overtime is not included with the total cost, labor costs are based on the average of 30 cities in North America, and location expenses are based on metropolitan areas where transportation is within a 20 mile radius of large cities (RSMeans v, vii).

The costs for the W-section beams are based on data from RSMeans. Costs for approximately 30 percent of the members listed in the AISC manual are given in RSMeans. An average cost per weight of steel is taken for specific ranges for which information is listed, and member weights calculated within those ranges are assigned the corresponding costs. A list of the weight ranges and costs applied to the beam weights are summarized in Table 1.

Steel Fabricators

While the steel fabricators interviewed were not at liberty to disclose specific details of their cost estimations, some were willing to provide a general breakdown of the total cost of materials and labor for a typical beam and truss used in the system.

Fabricators find the cost per foot of members from catalogs provided by the steel mills and add in labor costs, overhead and profit, etc. Mill prices for steel members average 40 cents per pound for common shapes to 50 cents per pound for less common shapes. W-shapes average 43 cents per pound, while HSS average about ten cents more per pound. The price increases for smaller sections. Heavier HSS and W-shapes tend to have similar costs per pound (Williams, Montgomery, Sprouse).

Truss cost estimations given by the steel fabricators are based on fully shop assembled trusses and current market conditions, including expenses for material, preparation of members (cutting, burning, fitting, handling, and priming), shop drawings, labor, overhead and profit, freight allowance, and installation. In general, the fabricators tended to find the cost of each of

Table 1: Average costs for W-section beam weights.

W-beam weight/ft	**Cost/lb**
0-10 plf	$2.0917
11-20 plf	$1.7053
21-30 plf	$1.4561
31-50 plf	$1.3441
51-90 plf	$1.1100
91-180 plf	$0.8419
191-270 plf	$0.8453
301-350 plf	$0.8548

these items in terms of the total weight of each truss. A reduction in cost is given for duplicate trusses. Shop drawing costs depend on the complexity of the details for the framing system. Freight costs are based on a shipment within 300 miles of the fabrication facility, with a maximum truck load of 45,000 lbs. (Turner, Sprouse, Williams).

Since the beam cost estimates given by the fabricators are comparable to the prices listed in RSMeans, Table 1 is used to calculate the cost of the beams.

The most applicable listing for trusses given in RSMeans is a “roof truss” with a minimum and maximum cost per ton of $3,475 and $4,425 respectively. John H. Ferguson, P.E., Sr. Manager of RSMeans/Reed Construction Data indicated that this number applies to typical trusses constructed of double angle or wide flange sections bolted together with gusset plates and that these costs can also be applied to floor systems. This particular cost estimate averages $1.98 per pound of truss. This estimate differs greatly from the estimates given by the steel fabricators interviewed. The average cost given by the fabricators for a truss consisting of W-shape chord members and double angle web members with half inch gusset plates and an average of four bolts per connection is $4.00 per pound of truss. For comparable HSS trusses, supporting the same load, material costs are approximately half as much as the latter, while labor costs are about 25 percent more for HSS.

Two HSS-truss drawings, as shown in Figure 11, were given to steel fabricators in order for them to provide a cost estimate breakdown per truss. The first is a truss in an 80’x80’ floor area, and the other is in a 40’x40’ floor area. The sections were chosen based on optimized weight. The floor load applied included a 100 psf live load and 20 psf dead load. The fabricators’ estimates are itemized in Table 2. Because a few of the fabricators were unable to disclose a complete itemization of their total numbers, some costs are guesstimated by the author. Through an analysis of the estimates provided, the cost of trusses has been assumed as $6.00/lb for calculations. The cost per pound for smaller trusses increases because the cost of small sections is more per pound than for larger sections. Cheaper costs per pound for heavier sections are compromised by more expensive shipping and handling costs and costs for splicing members if necessary for longer members. Therefore an average of $6.00/lb has been chosen.

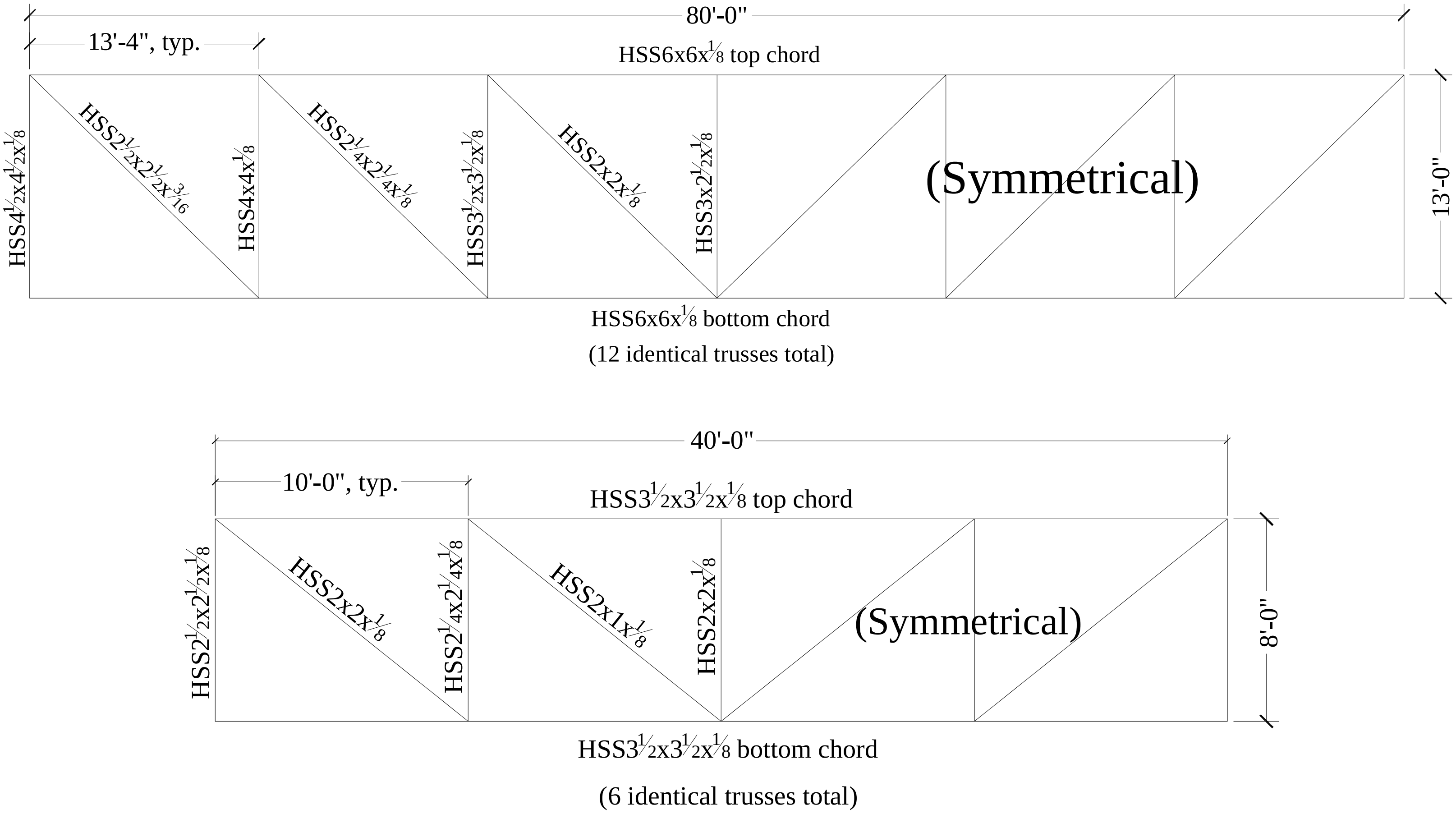


Figure 11: Two trusses analyzed by steel fabricators.

Table 2: Truss cost estimations given by steel fabricators.

	Fabricator 1		Fabricator 2		Fabricator 3	
	80'	**40'**	**80'**	**40'**	**80'**	**40'**
Materials ($/lb)	$1.29	$1.43	$0.55	$0.56	$1.08	$1.08
Labor ($/lb)	$1.34	$2.86	$1.11	$2.55	$1.59	$4.19
Shop drawings ($/lb)	$0.43	$1.34	$0.18	$0.64	$0.03	$0.06
Shipping ($/lb)	$0.22	$0.63	$1.41	$0.30	$0.34	$1.19
Overhead & profit ($/lb)	$2.30	$4.80	$0.25	$0.49	$0.41	$0.96
Total ($/truss)	$12,942.50	$5,686.00	$12,330.00	$3,550.00	$10,782.26	$5,413.51

	Fabricator 4		Fabricator 5	
	80'	**40'**	**80'**	**40'**
Materials ($/lb)	$0.67	$0.74	$0.74	$0.81
Labor ($/lb)	$1.06	$2.27	$1.12	$2.39
Shop drawings ($/lb)	$0.69	$2.13	$0.32	$0.99
Shipping ($/lb)	$0.08	$0.23	$0.07	$0.21
Overhead & profit ($/lb)	$1.05	$2.18	$0.90	$1.86
Total ($/truss)	$10,014.00	$4,399.43	$11,000.00	$4,832.61

CHAPTER 6

GENETIC ALGORITHMS

Procedure

Developed by John Holland, genetic algorithms, a series of evolutionary algorithms, are used to improve on previous solutions found during the search process. This *meta-heuristic* search method incorporates the concept of natural selection and evolution in the random search for better alternatives. This method may not always find the exact optimum, but can quickly find good quality solutions. The relative fitness of a particular solution determines whether it should either be discarded or incorporated in the search for an improved answer.

A *heuristic* algorithm searches for a better solution in a region surrounding an existing solution and repeats the process if a better solution has been found. Edgar et al. (2001) defined a *meta-heuristic* algorithm as an improved heuristic algorithm, in that it modifies its current search method so as to not become trapped in a local optimum.

Combining two superior solutions from the current population and using crossover and mutation progressions to produce better solutions for the new population emulates reproduction in evolutionary systems. Crossover is non-deterministic in nature, in that random points in the parent chromosomes are exchanged. Mutation is also non-deterministic since random genes will change to any random value. Selection is carried out to find an optimum of a particular strategy, while crossover and mutation expand the search process (Reed 187-189).

Chelouah and Siarry (2001) explain the step-by-step process of a genetic algorithm. The generation of the size of the initial population of chromosomes, variation of the population size, initial mutation probability value, and rule of decreasing crossover and mutation steps precede the search process. From this *initialization* step, *diversification* is performed where the initial population is selected and from that, productions of subsequent, smaller populations are chosen through selection, crossover, and mutation. Superior sub-populations are found through *intensification* on better areas of larger populations. Diversification is repeated, and the algorithm becomes recursive. The population size remains constant throughout the entire process. Several different population sizes may be tested in order to find the most optimal solution.

Ortiz et al. (2004) suggested an approach to optimize the search parameters of a GA, particularly under multiple response conditions of varying complexities to enhance the GA's performance. Such parameters included in the study were the population size, crossover and mutation probabilities, parent-offspring ratio, and selection and mutation types. Because several factors must be adjusted for the operation of a genetic algorithm, it can be difficult to distinguish how each parameter affects the search process. Therefore, it becomes beneficial to the user to have a method for which each value is determined, especially for more complex problems containing a large number of variables and/or outcomes. By specifying appropriate numbers for each of the search operators, the GA is less likely to converge to a local optimum solution.

Application to Truss Optimization

With application to truss optimization, an algorithm starts with an initial population of "n" individual chromosomes corresponding to different solutions to the problem. The makeup of each chromosome is composed of three segments: number of purlins, number of trusses, and truss height. Every solution has unique values for each of these segments.

Operational adjustments of the algorithm are carried out after several generations without improvement of the objective function. As the size of the search space is reduced, so are the crossover and mutation steps (Chelouah et al.).

CHAPTER 7

EVOLVER

Background

Evolver is an add-in for Microsoft Excel that finds better solutions to a problem by adjusting variables through genetic algorithms. Finding an optimization program that quickly provides a solution to the problem is as important as creating an Excel program that immediately gives the cost of a particular system for any three variables. Working with the writer's Excel cost program, Evolver rapidly adjusts the three variables to progress to the most optimal solution. This eliminates guesswork by the user and saves a countless amount of manual optimization time. With Evolver, constraint formulas are not required to be "compromised" as they are with simpler evaluation programs such as Excel's Solver add-in (Palisade 26). For example, Solver has trouble enforcing the *integer* constraint and can easily become trapped in local optimum solutions. While Evolver contains the "power and flexibility" to evaluate complex problems, it is easy to operate, using a similar format as Solver (Palisade 27). Also, simplifying the problem from real value coding to binary coding is not necessary as with other GA software programs.

Analysis

The cost of each member of the floor system is summed to obtain a total cost of the floor system. Evolver allows the user to find the minimum value for the cost by automatically adjusting the cell values for the number of beams, number of truss girders, and truss height.

The "recipe" solving method is used to adjust each cell within its own particular cell range. This method is a type of GA with a specific optimized selection, crossover, and mutation routine. Each of the three variables is viewed as an ingredient in a recipe, where they can be altered independently of one another. In order to avoid interpolation and to use only discrete variables, integer values are selected for the process (Palisade 101).

The population size represents the number of organisms to be stored in Evolver's memory at any given time. The chosen population size, *six*, is equal to twice the number of variables, recommended by Mayer, et al. (2001) for real-value encoded problems.

The given range for each adjustable cell is a *hard constraint* that is strictly enforced. A broad range has been selected for the variables to provide Evolver a large search space. A *soft constraint* permits leniency of the given limitations in anticipation of improvement of fitness in the results. Although Evolver will not converge to an infeasible solution, it evaluates infeasible solutions in order to search for feasible areas (Palisade 108, 116).

Additional constraints placed on the solution include the selection of an odd number of beams, the 30 degree minimum branch member orientation from both horizontal and vertical truss members, and the total load and live load truss deflection limitations. These are all taken as hard constraints.

The crossover rate indicates the probability that offspring will be comprised of a combination of information from their parents. Crossover rates can vary between 0.01 and 1, where 1 signifies that no crossover will take place (Palisade 110, 111). A cross over rate of 0.5 is used at the beginning of the search process, and it is increased to 0.85 after several generations with no change in the best solution.

The mutation rate specifies the probability that offspring will include some random variables. Mutation rates can vary between 0 and 1, where a rate of 1 indicates that all variables will be totally random (Palisade 111). A moderately high mutation rate of .4 is used at the beginning of the search process, and a lower mutation rate of .1 is used when the cross over rate is lowered. By lowering both the crossover and mutation rates, this forces the GA to concentrate on a specific area of the search space, permitting less diversity in the chromosome.

An example of a typical graph produced by Evolver is shown in Figure 12. The best solution found thus far for each trial is plotted. This particular graph is for a Howe truss in a 40'x40' floor area. The initial solution was about $52,000. After a period of time with no change in the best solution (50–100 trials), the crossover and mutation rates were changed.

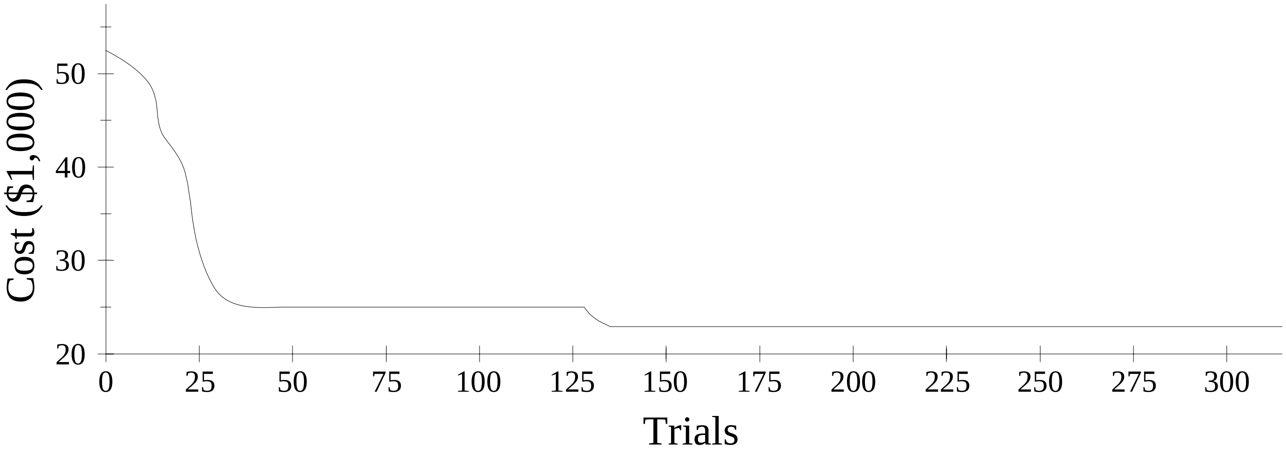


Figure 12: Plot of Evolver's progression to most optimal solution.

Finally, after several more generations with no change in the most optimal solution, $23,000 was taken as the final solution. It is not possible to tell whether a GA has found the global optimum answer to a problem, but it is possible for it to have found a much improved answer. This took only about three minutes for Evolver to optimize.

CHAPTER 8

RESULTS

Three different floor areas are optimized for weight and cost for both Pratt and Howe truss topologies as shown in Tables 3 and 4. Because smaller sections cost more per pound than larger sections, the smallest weight floor system does not always correspond to the least cost floor system. The optimization summaries given by Evolver for these tables are shown in Appendix C.

Figure 13 shows the differences in cost for trusses versus beams spanning in the short direction for different rectangular floor areas. The floor frame costs less when trusses run in the shorter direction.

Figures 14 – 16 shows the costs of floor framing with Howe versus Pratt trusses for different loads and floor area dimensions. Framing using Howe trusses costs less for longer spans, whereas for shorter spans, Pratt trusses govern.

Figure 17 shows a sensitivity analysis of the variation in total floor system cost with the average cost per pound of truss given by each of the steel fabricators for a 60’x60’ floor area with a 100 psf live load and 20 psf dead load applied. Howe trusses are used in the design. The average cost per pound of truss from each fabricator is shown on each bar of the graph. These are compared to the cost per pound of truss chosen for the analysis, $6.00/lb of truss. There was not a trend in similar costs for the same state or even similar facility sizes. Therefore, the costs are ordered by fabricator. It can be seen that for even a small difference in cost per pound of truss by only 15 cents between Fabricator 5 and the chosen cost, a large difference of $2,275 can be seen in the total cost of the floor system. Overall, the total costs varied between $86,050 and $111,500, with the total cost using $6.00/lb of truss being $98,335. It is recommended for the designer to compare prices with several fabricators in order to save money in construction costs.

Table 3: Pratt truss optimum weight and cost values for different floor areas.

	Pratt - Weight Optimized		
	100'x100' Area	**80'x80' Area**	**40'x40' Area**
Number of Beams	9	7	5
Number of Trusses	12	12	6
Height of trusses	14	13	8
Best Value Found (lbs)	61,226.18	30,604.45	5,061.84
Total Floor Cost	$328,499.96	$164,940.67	$23,697.47

	Pratt - Cost Optimized		
	100'x100' Area	**80'x80' Area**	**40'x40' Area**
Number of Beams	9	7	5
Number of Trusses	12	11	5
Height of trusses	14	15	7
Total Weight (lbs)	61,226.18	31,171.01	5,210.59
Best Value Found	$328,499.96	$162,848.32	$22,628.62

Table 4: Howe truss optimum weight and cost values for different floor areas.

	Howe - Weight Optimized		
	100'x100' Area	**80'x80' Area**	**40'x40' Area**
Number of Beams	11	7	5
Number of Trusses	11	12	6
Height of trusses	15	13	8
Best Value Found (lbs)	60,668.99	30,617.72	5,256.09
Total Floor Cost	$316,521.95	$165,020.30	$24,862.93

	Howe - Cost Optimized		
	100'x100' Area	**80'x80' Area**	**40'x40' Area**
Number of Beams	11	9	5
Number of Trusses	11	10	5
Height of trusses	15	14	10
Total Weight (lbs)	60,668.99	31,928.89	5,370.08
Best Value Found	$316,521.95	$160,487.66	$23,585.60

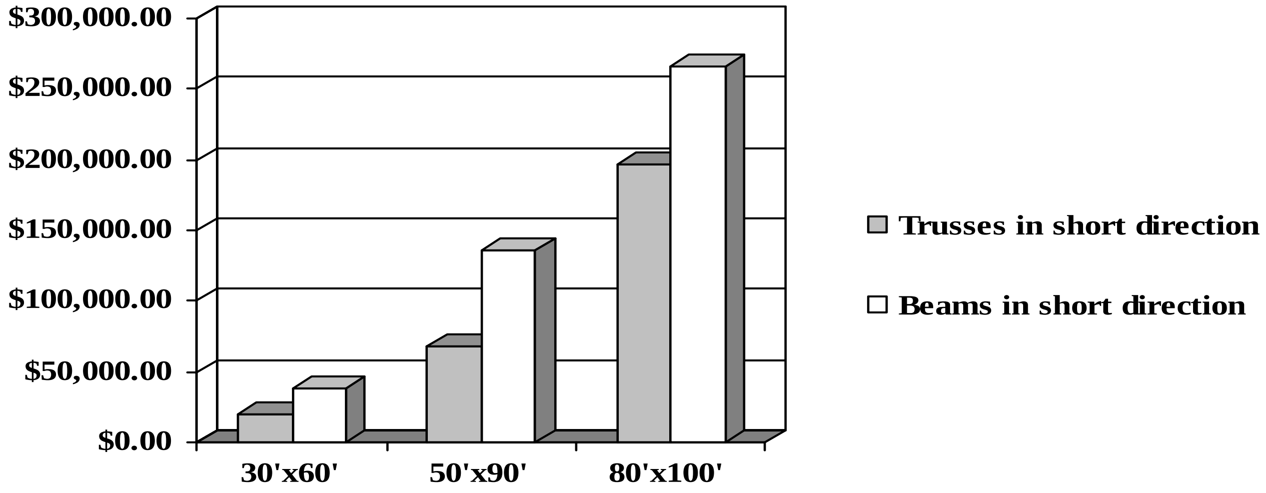


Figure 13: Rectangular floor area analysis.

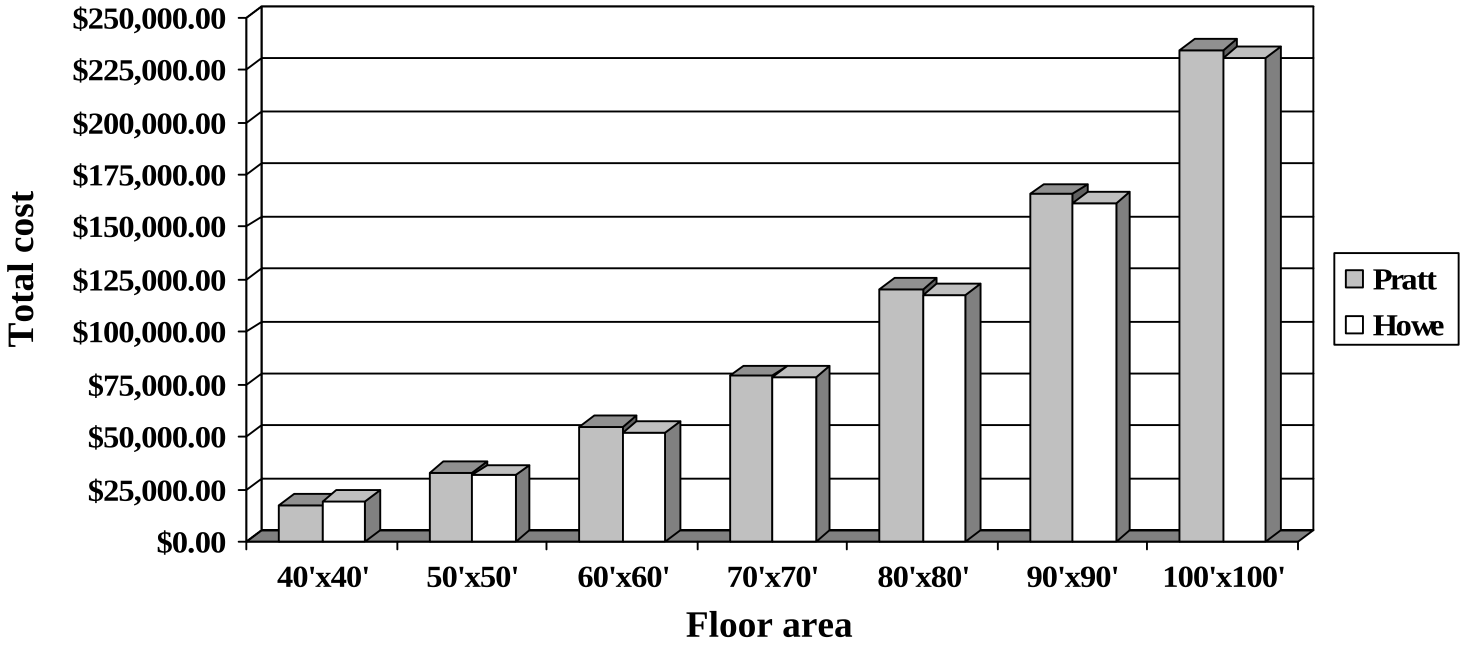


Figure 14: Optimized floor framing costs using Pratt and Howe trusses, 60 psf load.

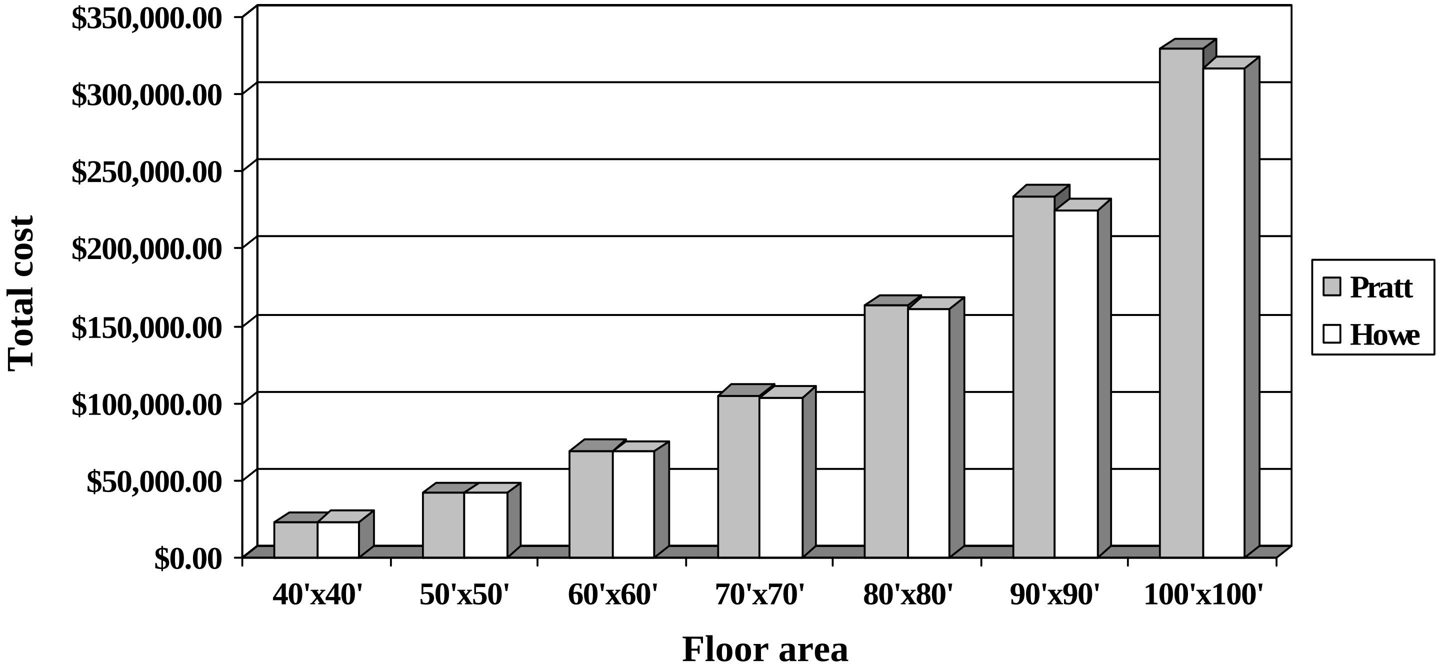


Figure 15: Optimized floor framing costs using Pratt and Howe trusses, 120 psf load.

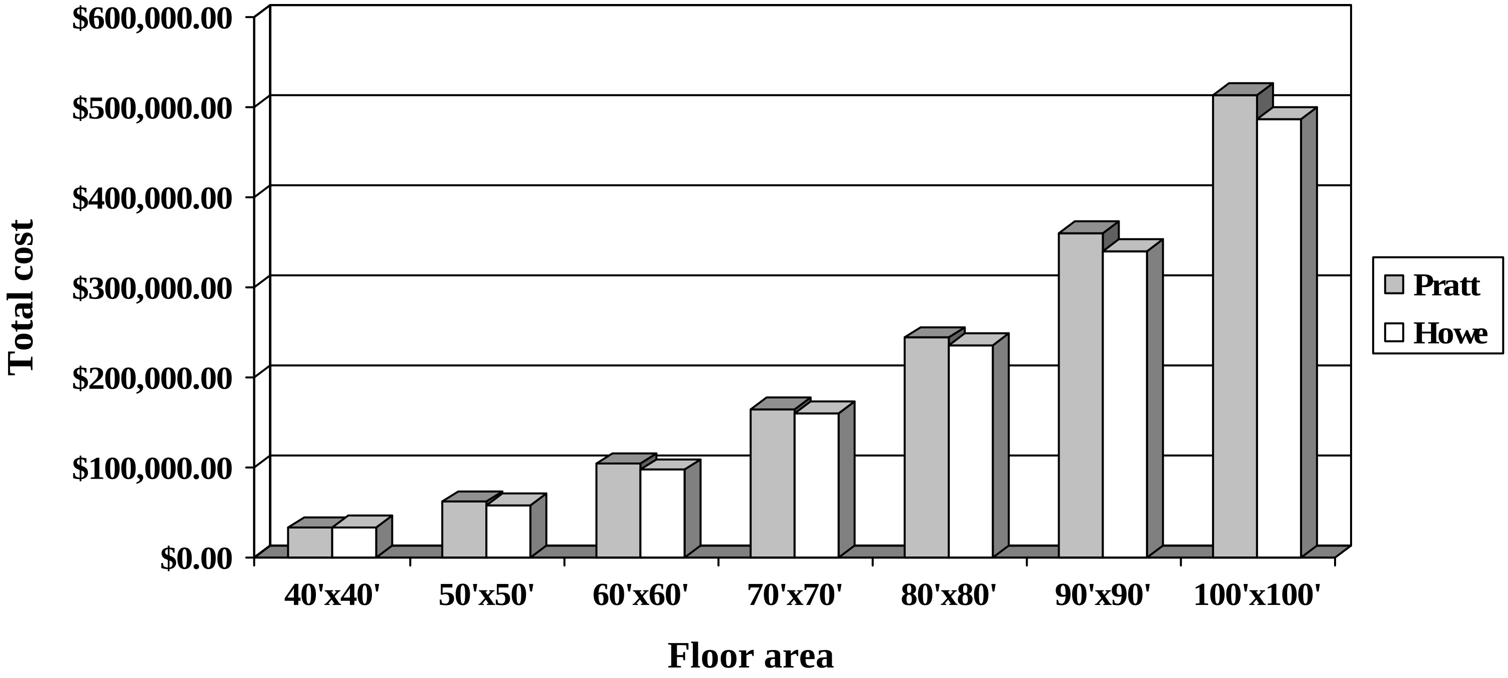


Figure 16: Optimized floor framing costs using Pratt and Howe trusses, 240 psf load.

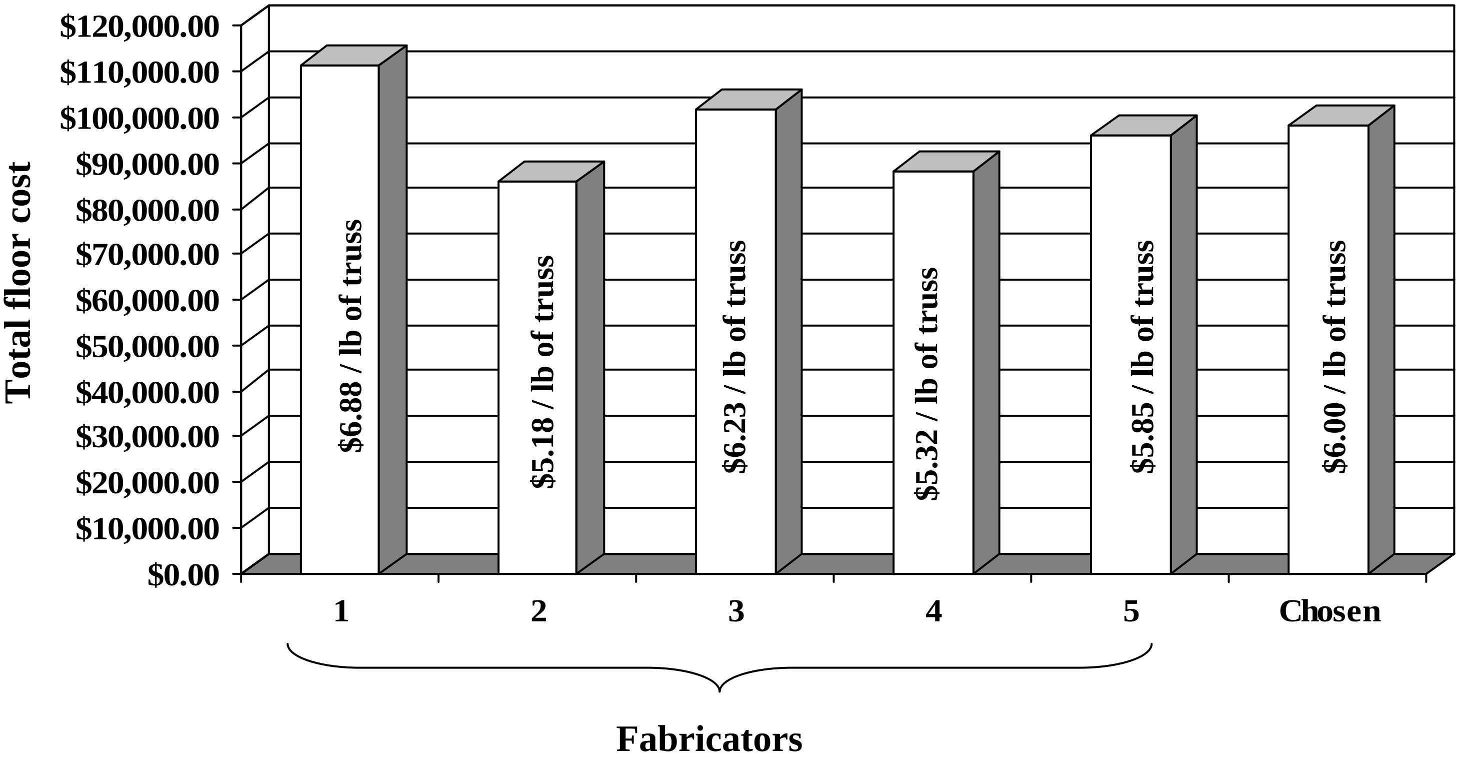


Figure 17: Cost sensitivity analysis of average costs/lb given by fabricators, 60'x60' floor area.

CONCLUSION

A procedure for automatically choosing the most efficient floor framing for any loading condition or floor area becomes invaluable to an engineer. This cuts out the guesswork of having to calculate different trials by hand and taking time to look up values in tables, or developing structural models in design programs such as STAAD or SAP for each floor frame considered. A spreadsheet can be developed that includes member shape selection tables within the program to automatically assign members to any floor system, similar to the one created for this study. Evolver is an optimization tool that can be used with an Excel spreadsheet to minimize the cost of a floor frame. After inputting the parameters, it should only take about five minutes to run the analysis and find a much improved solution. Where cost is determined as a function of the weight of the members, it is not necessary to detail the connections of the system to come up with a total estimate. This method optimizes the beam and truss configuration while providing a quick and useful estimate of the floor frame. This technique can be functional to both professionals and students in the fields of engineering and optimization.

Size, configuration, and topology optimization of the trusses, size optimization of the beams, and the optimization of spacings of both beams and trusses is demonstrated in this solving method. Cost data was compiled from *Means Building Construction Cost Data* and interviews with steel fabricators to include expenses for material, labor, framing connections, equipment, and overhead and profit. This is a basic procedure that can be customized for the cost optimization of any size floor or flat roof area that may use different truss girder topologies, beam sections, and connections.

Recommendations

In order to avoid the expensive fabrication of stiffeners for reinforcement in the chords, it is suggested by AISC to design thicker chords with thinner branches connecting to them. Because the higher forces in the truss concentrate in the middle of the top and bottom chords, and each chord is designed for the highest force throughout its length, the chords will usually already be designed larger than the web members. If the corresponding chord area matches a section with a narrow width, the chord members may be chosen wider than the broadest web member while satisfying all design constraints. This can be done through using the Excel "SMALL" command, where the next smallest area would be chosen, and the width of that section could be compared to the broadest web member.

Additional tables may be created in order to accommodate other truss topologies for the analysis. The choices of sections may be designated to commonly used shapes if desired. Grouping of web or chord members for the purpose of using fewer sections for each truss may also reduce cost.

Suggestions for Further Research

This optimization method may be applied to other structural member shapes by setting up flexural, compression, or tension tables with the desired shape's properties and using equations that satisfy the shape's associated strength and serviceability limit states.

Assumptions made in this thesis may further be analyzed and validated, such as typical connections used in the frame and the correlating costs. Furthermore, the connections could be optimized in terms of expense. Optimization of the cost of connections could be done where a complication factor could be employed as a soft constraint in a penalty function with Evolver, corresponding to the complexity of connections. This factor may correspond to the member sizes, number of members connecting at each joint, and angle of connectivity.

Specific material, labor, and equipment costs could be broken down into detail. Also, fabrication, shipping, handling, and constructing times could be evaluated, and cost could be proportioned in terms of hourly rates.

APPENDIX A

MAXIMUM SLAB SPANS OF VARIOUS THICKNESSES

Maximum Slab Spans of Various Thicknesses

(MathCad Worksheet)

For one-way action:

Following Nilson, Ex. 13.1:

$psf := \frac{lbf}{ft^2}$

$pcf := \frac{lbf}{ft^3}$

$plf := \frac{lbf}{ft}$

$\beta_1 := .85$ (ACI 10.2.7.3)

$f'_c := 3ksi$

$F_y := 60ksi$

$\varepsilon_u := .002$ (ACI 10.3.3)

$E := 29000ksi$

$\varepsilon_t := \max\left(\frac{F_y}{E}, .004\right)$ $\varepsilon_t = 4 \times 10^{-3}$ (ACI 10.3.5)

$\rho_{max} := .85\beta_1 \cdot \frac{f'_c}{F_y} \cdot \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_t}$ $\rho_{max} = 0.012$ (Nilson, Eq. 3.30*b*)

$b := 1ft$ (Unit section of slab)

Slab thickness:

$h := 4in$

Cover:

$c := .75in$ (ACI 7.7.1 (c))

Bar diameter:

$d_b := \frac{5}{8}in$ (Assumption)

Effective depth:

$d := h - c - d_b$ $d = 2.625\,in$

$\phi_b := .9$ (ACI 9.3.2.1)

$R := \rho_{max} \cdot F_y \cdot \left(1 - .59\frac{\rho_{max} \cdot F_y}{f'_c}\right)$ $R = 619.839\ psi$ (Nilson, Eq. 3.36)

$\phi M_n := \phi_b \cdot R \cdot b \cdot d^2$ $\quad \phi M_n = 3.844\,\text{kip}\cdot\text{ft}$ (Nilson, Eq. 3.39)

$\gamma_c := 115\text{pcf}$ (Lightweight concrete)

Service dead load applied to slab:

$q_{DL} := \gamma_c \cdot h + 20\text{psf}$ $\quad q_{DL} = 58.333\,\text{psf}$

$w_{DL} := q_{DL} \cdot b$ $\quad w_{DL} = 58.333\,\text{plf}$

Service live load applied to slab:

$q_{LL} := 100\text{psf}$ (FBC Table1607.1)

$w_{LL} := q_{LL} \cdot b$ $\quad w_{LL} = 100\,\text{plf}$

$w := w_{DL} + w_{LL}$ $\quad w = 158.333\,\text{plf}$

$l_1 := \sqrt{\frac{8\phi M_n}{w}}$ $\quad l_1 = 13.936\text{ ft}$

$l_2 := 28 \cdot h$ $\quad l_2 = 9.333\text{ ft}$ (ACI Table 9.5 (a))

$l := \min(l_1, l_2)$ $\quad l = 9.333\text{ ft}$ (Maximum span of 4" slab)

Maximum span of 4" slab: 9'-4"
Maximum span of 5" slab: 11'-8"
Maximum span of 6" slab: 14'
Maximum span of 7" slab: 16'-4"
Maximum span of 8" slab: 18'-8"
Maximum span of 9" slab: 21'
Maximum span of 10" slab: 23'-4"
Maximum span of 11" slab: 25'-8"
Maximum span of 12" slab: 28'

The maximum slab span increases by 2'-4" for every inch of slab thickness.

APPENDIX B

STEP-BY-STEP FLOOR FRAMING CALCULATIONS

Step-by-Step Floor Framing Calculations

(MathCad Worksheet)

$$\text{pcf} := \frac{\text{lbf}}{\text{ft}^3}$$

$$\text{psf} := \frac{\text{lbf}}{\text{ft}^2}$$

$$\text{plf} := \frac{\text{lbf}}{\text{ft}}$$

primary parameters:

$n_b := 9$ (odd)

$n_t := 8$

$h := 13\text{ft}$

secondary parameters:

$x_b := 80\text{ft}$ (dimension of floor system in direction of beam span)

$x_t := 80\text{ft}$ (dimension of floor system in direction of truss span)

beams:

$$s_b := \frac{x_t}{n_b - 1} \qquad s_b = 10\,\text{ft}$$

$$\max\left(\frac{\sqrt{3}}{3} - \frac{h}{s_b}, \text{if}\left(\frac{h}{s_b} - \sqrt{3} > 0, \frac{h}{s_b} - \sqrt{3}, 0\right)\right) = 0 \qquad \text{(angle check - must be 0)}$$

$$t := \text{if}\left[s_b \le 9\text{ft} + 4\text{in}, 4\text{in}, 4\text{in} + \text{ceil}\left[\frac{s_b - (9\text{ft} + 4\text{in})}{2\text{ft} + 4\text{in}}\right]\text{in}\right] \qquad t = 5\,\text{in}$$

$\gamma_c := 115\text{pcf}$

$q_{DL} := \gamma_c \cdot t + 20\text{psf} \qquad q_{DL} = 67.917\,\text{psf}$

$w_{DL} := q_{DL} \cdot s_b \qquad w_{DL} = 679.167\,\text{plf}$

$q_{LL} := 100\text{psf}$

$w_{LL} := q_{LL} \cdot s_b \qquad w_{LL} = 1 \times 10^3\,\text{plf}$

$w := w_{LL} + w_{DL} \quad w = 1.679 \times 10^3\,\text{plf}$

$s_t := \dfrac{x_b}{n_t - 1} \quad s_t = 11.429\,\text{ft}$

$M := \dfrac{w \cdot s_t^{\,2}}{8} \quad M = 27.415\,\text{kip}\cdot\text{ft}$

$\phi_b := .9$

$M_n := \dfrac{M}{\phi_b} \quad M_n = 30.461\,\text{kip}\cdot\text{ft}$

$F_y := 50\text{ksi}$

$Z_{x.r} := \dfrac{M_n}{F_y} \quad Z_{x.r} = 7.311\,\text{in}^3$(required)

$\Delta_{TL} := \dfrac{s_t}{240} \quad \Delta_{TL} = 0.571\,\text{in}$

$\Delta_{LL} := \dfrac{s_t}{360} \quad \Delta_{LL} = 0.381\,\text{in}$

$E := 29000\text{ksi}$

$I_{x.r} := \max\left(\dfrac{5w \cdot s_t^{\,4}}{384E \cdot \Delta_{TL}}, \dfrac{5w_{LL} \cdot s_t^{\,4}}{384E \cdot \Delta_{LL}}\right) \quad I_{x.r} = 38.894\,\text{in}^4$

"beams" table:

for each W-shape in "beams" table:

W10x12 for example:

$I_x := 53.8\text{in}^4 \quad Z_x := 12.6\text{in}^3 \quad A := 3.54\text{in}^2$

$A_1 := \max\left(\text{if}\left(I_x \geq I_{x.r}, A, 9999\text{in}^2\right), \text{if}\left(Z_x \geq Z_{x.r}, A, 9999\text{in}^2\right)\right) \quad A_1 = 3.54\,\text{in}^2$

$A_g = \min\left(A_1\right)$for all W-shapes

$A_g = 3.54\,\text{in}^2$

$w_{sw} := 3.4\dfrac{\text{psi}}{\text{ft}}A_g \quad w_{sw} = 12.036\,\text{plf}$ (self weight)

$w_i := w + w_{sw} \quad w_i = 1.691 \times 10^3\,\text{plf}$ (total load on each interior beam)

$M := \frac{w_i \cdot {s_t}^2}{8}$ $M = 27.611\,\text{kip}\cdot\text{ft}$

$M_n := \frac{M}{\phi_b}$ $M_n = 30.679\,\text{kip}\cdot\text{ft}$

$Z_{x.r} := \frac{M_n}{F_y}$ $Z_{x.r} = 7.363\,\text{in}^3$

$I_{x.r} := \max\left(\frac{5w_i \cdot {s_t}^4}{384E \cdot \Delta_{TL}}, \frac{5w_{LL} \cdot {s_t}^4}{384E \cdot \Delta_{LL}}\right)$ $I_{x.r} = 39.173\,\text{in}^4$

"beams" table:

for each W-shape in "beams" table:

W10x12 for example:

$I_x := 53.8\text{in}^4$ $Z_x := 12.6\text{in}^3$ $A := 3.54\text{in}^2$

$A_2 := \max\left(\text{if}\left(I_x \ge I_{x.r}, A, 9999\text{in}^2\right), \text{if}\left(Z_x \ge Z_{x.r}, A, 9999\text{in}^2\right)\right)$ $A_2 = 3.54\,\text{in}^2$

$A_g = \min\left(A_2\right)$ for all W-shapes

$A_g = 3.54\,\text{in}^2$

$w_{sw} := 3.4\frac{\text{psi}}{\text{ft}}A_g$ $w_{sw} = 12.036\,\text{plf}$

$w_i := w + w_{sw}$ $w_i = 1.691 \times 10^3\,\text{plf}$

$w_e := \frac{w}{2} + w_{sw}$ $w_e = 851.619\,\text{plf}$ (total load on each end beam)

trusses - total load applied:

$P_i := w_i \cdot s_t$ $P_i = 1.933 \times 10^4\,\text{lbf}$ (load from interior beams applied to trusses)

$P_e := w_e \cdot s_t$ $P_e = 9.733 \times 10^3\,\text{lbf}$ (load from end beams applied to trusses)

$V := \frac{P_i \cdot \left(n_b - 2\right) + 2P_e}{2}$ $V = 7.738 \times 10^4\,\text{lbf}$

$K_b := .75$ (effective length factor for branch members in compression)

$K_c := .9$ (effective length factor for chord members in compression)

$F_y := 46\text{ksi}$

Howe truss:

internal force in each truss member due to total load applied to truss:

$T_1 := 0$

$C_2 := 0$

$C_3 := P_e \quad C_3 = 9.733\,\text{kip}$

$$C_4 := \frac{\left(V - C_3\right)\cdot\sqrt{s_b^{\,2} + h^2}}{h} \quad C_4 = 85.347\,\text{kip}$$

$$T_5 := \frac{C_4\cdot s_b}{\sqrt{s_b^{\,2} + h^2}} \quad T_5 = 52.037\,\text{kip}$$

for each additional double-panel set beyond initial 2-panel truss:

$i := 1$

$$C_{2+4\cdot i} := \text{if}\left[n_b \le 3 + 2(i-1), 0, T_{5+4(i-1)}\right] \quad C_{2+4\cdot i} = 52.037\,\text{kip}$$

$$T_{3+4\cdot i} := \text{if}\left[n_b \le 3 + 2(i-1), 0, \frac{C_{4\cdot i}\cdot h}{\sqrt{s_b^{\,2} + h^2}} - P_i\right] \quad T_{3+4\cdot i} = 48.32\,\text{kip}$$

$$C_{4+4\cdot i} := \text{if}\left[n_b \le 3 + 2(i-1), 0, \frac{T_{3+4\cdot i}\cdot\sqrt{s_b^{\,2} + h^2}}{h}\right] \quad C_{4+4\cdot i} = 60.962\,\text{kip}$$

$$T_{5+4\cdot i} := \text{if}\left[n_b \le 3 + 2(i-1), 0, T_{1+4\cdot i} + \frac{C_{4+4\cdot i}\cdot s_b}{\sqrt{s_b^{\,2} + h^2}}\right] \quad T_{5+4\cdot i} = 89.206\,\text{kip}$$

$i := i + 1 \quad i = 2$

$$C_{2+4\cdot i} := \text{if}\left[n_b \le 3 + 2(i-1), 0, T_{5+4(i-1)}\right] \quad C_{2+4\cdot i} = 89.206\,\text{kip}$$

$$T_{3+4\cdot i} := \text{if}\left[n_b \le 3 + 2(i-1), 0, \frac{C_{4\cdot i}\cdot h}{\sqrt{s_b^{\,2} + h^2}} - P_i\right] \quad T_{3+4\cdot i} = 28.992\,\text{kip}$$

$$C_{4+4\cdot i} := \text{if}\left[n_b \le 3 + 2(i-1), 0, \frac{T_{3+4\cdot i}\cdot\sqrt{s_b^{\,2} + h^2}}{h}\right] \quad C_{4+4\cdot i} = 36.577\,\text{kip}$$

$$T_{5+4\cdot i} := \text{if}\left[n_b \le 3 + 2(i-1), 0, T_{1+4\cdot i} + \frac{C_{4+4\cdot i}\cdot s_b}{\sqrt{s_b^2 + h^2}}\right] \quad T_{5+4\cdot i} = 111.508\,\text{kip}$$

$i := i + 1 \quad i = 3$

$$C_{2+4\cdot i} := \text{if}\left[n_b \le 3 + 2(i-1), 0, T_{5+4(i-1)}\right] \quad C_{2+4\cdot i} = 111.508\,\text{kip}$$

$$T_{3+4\cdot i} := \text{if}\left[n_b \le 3 + 2(i-1), 0, \frac{C_{4\cdot i}\cdot h}{\sqrt{s_b^2 + h^2}} - P_i\right] \quad T_{3+4\cdot i} = 9.664\,\text{kip}$$

$$C_{4+4\cdot i} := \text{if}\left[n_b \le 3 + 2(i-1), 0, \frac{T_{3+4\cdot i}\cdot\sqrt{s_b^2 + h^2}}{h}\right] \quad C_{4+4\cdot i} = 12.192\,\text{kip}$$

$$T_{5+4\cdot i} := \text{if}\left[n_b \le 3 + 2(i-1), 0, T_{1+4\cdot i} + \frac{C_{4+4\cdot i}\cdot s_b}{\sqrt{s_b^2 + h^2}}\right] \quad T_{5+4\cdot i} = 118.942\,\text{kip}$$

required radius of gyration for zero-force members:

$$r_{y_1} := \frac{K_b\cdot h}{300} \quad r_{y_1} = 0.39\,\text{in}$$

$$r_{y_2} := \frac{K_c\cdot s_b}{200} \quad r_{y_2} = 0.54\,\text{in}$$

required lengths and effective lengths for other members:

$$KL_3 := \text{Ceil}\left(K_b\cdot h, 1\text{ft}\right) \quad KL_3 = 10\,\text{ft}$$

$$KL_4 := \text{Ceil}\left(K_b\cdot\sqrt{s_b^2 + h^2}, 1\text{ft}\right) \quad KL_4 = 13\,\text{ft}$$

$$L_5 := \text{Ceil}\left(s_b, 1\text{ft}\right) \quad L_5 = 10\,\text{ft}$$

$i := 1$

$$KL_{2+4\cdot i} := \text{if}\left(C_{2+4\cdot i} = 0, 0, \text{Ceil}\left(K_c\cdot s_b, 1\text{ft}\right)\right) \quad KL_{2+4\cdot i} = 9\,\text{ft}$$

$$L_{3+4\cdot i} := \text{if}\left(T_{3+4\cdot i} = 0, 0, \text{Ceil}(h, 1\text{ft})\right) \quad L_{3+4\cdot i} = 13\,\text{ft}$$

$$KL_{4+4\cdot i} := \text{if}\left(C_{4+4\cdot i} = 0, 0, \text{Ceil}\left(K_b\cdot\sqrt{s_b^2 + h^2}, 1\text{ft}\right)\right) \quad KL_{4+4\cdot i} = 13\,\text{ft}$$

$L_{5+4\cdot i} := \text{if}\left(T_{5+4\cdot i} = 0, 0, \text{Ceil}\left(s_b, 1\text{ft}\right)\right) \quad L_{5+4\cdot i} = 10\,\text{ft}$

$i := i + 1 \quad i = 2$

$KL_{2+4\cdot i} := \text{if}\left(C_{2+4\cdot i} = 0, 0, \text{Ceil}\left(K_c \cdot s_b, 1\text{ft}\right)\right) \quad KL_{2+4\cdot i} = 9\,\text{ft}$

$L_{3+4\cdot i} := \text{if}\left(T_{3+4\cdot i} = 0, 0, \text{Ceil}(h, 1\text{ft})\right) \quad L_{3+4\cdot i} = 13\,\text{ft}$

$KL_{4+4\cdot i} := \text{if}\left(C_{4+4\cdot i} = 0, 0, \text{Ceil}\left(K_b \cdot \sqrt{{s_b}^2 + h^2}, 1\text{ft}\right)\right) \quad KL_{4+4\cdot i} = 13\,\text{ft}$

$L_{5+4\cdot i} := \text{if}\left(T_{5+4\cdot i} = 0, 0, \text{Ceil}\left(s_b, 1\text{ft}\right)\right) \quad L_{5+4\cdot i} = 10\,\text{ft}$

$i := i + 1 \quad i = 3$

$KL_{2+4\cdot i} := \text{if}\left(C_{2+4\cdot i} = 0, 0, \text{Ceil}\left(K_c \cdot s_b, 1\text{ft}\right)\right) \quad KL_{2+4\cdot i} = 9\,\text{ft}$

$L_{3+4\cdot i} := \text{if}\left(T_{3+4\cdot i} = 0, 0, \text{Ceil}(h, 1\text{ft})\right) \quad L_{3+4\cdot i} = 13\,\text{ft}$

$KL_{4+4\cdot i} := \text{if}\left(C_{4+4\cdot i} = 0, 0, \text{Ceil}\left(K_b \cdot \sqrt{{s_b}^2 + h^2}, 1\text{ft}\right)\right) \quad KL_{4+4\cdot i} = 13\,\text{ft}$

$L_{5+4\cdot i} := \text{if}\left(T_{5+4\cdot i} = 0, 0, \text{Ceil}\left(s_b, 1\text{ft}\right)\right) \quad L_{5+4\cdot i} = 10\,\text{ft}$

member areas:

"T" (tension) table:

for each HSS-shape in "T" table:

HSS1.25x1.25x1/8 for example:

$r_{y_0} := .454\text{in} \quad A_g := .492\text{in}^2$

$A := \text{if}\left(r_{y_0} \geq r_{y_1}, A_g, 99\text{in}^2\right) \quad A = 0.492\,\text{in}^2$

$A_{g_1} = \min(A)$ for all HSS-shapes

$A_{g_1} = 0.492\,\text{in}^2$

"C" (compression) table:

for each HSS-shape in "C" table:

HSS1.5x1.5x1/8 for example:

$r_{y_0} := .557\text{in} \quad A_{g_0} := .608\text{in}^2$

$A := \mathrm{if}\left(r_{y_0} \ge r_{y_2}, A_{g_0}, 99\mathrm{in}^2\right) \quad A = 0.608\ \mathrm{in}^2$

$A_{g_2} = \min(A)$ for all HSS-shapes)

$A_{g_2} := \mathrm{if}\left(n_b = 3, A_{g_2}, 0\right) \quad A_{g_2} = 0\ \mathrm{in}^2$

"C" (compression) table:

for each HSS-shape in "C" table:

HSS2.25x2.25x1/8 for example:

$r_{y_0} := .863\mathrm{in} \quad A_{g_0} := .956\mathrm{in}^2 \quad t := \frac{1}{8}\mathrm{in} \quad d_n := 2.25\mathrm{in}$ (nominal depth)

$b := d_n - 3t \quad b = 1.875\ \mathrm{in}$

$\lambda := \frac{b}{t} \quad \lambda = 15$

$\phi_c := .85$

for each effective length (similar to AISC Table 4-6):

$KL_3 = 10\ \mathrm{ft}$ for example

$\lambda_c := \frac{KL_3}{r_{y_0}\pi}\sqrt{\frac{F_y}{E}} \quad \lambda_c = 1.763$

$Q := 1$ (assumption)

$$F_{cr} := \mathrm{if}\left(\lambda_c \cdot \sqrt{Q} \le 1.5, Q \cdot .658^{Q \cdot \lambda_c^2} F_y, \frac{.877}{\lambda_c^2} F_y\right) \quad F_{cr} = 12.982\ \mathrm{ksi}$$

$f := \phi_c \cdot F_{cr} \quad f = 1.104 \times 10^4\ \mathrm{psi}$

$$b_e := \mathrm{if}\left[\lambda \ge 1.4\sqrt{\frac{E}{f}}, 1.91t\sqrt{\frac{E}{f}}\left(1 - \frac{.381}{\lambda}\sqrt{\frac{E}{f}}\right), b\right] \quad b_e = 1.875\ \mathrm{in}$$

$A_e := A_{g_0} - 4\left(b - b_e\right)t \quad A_e = 0.956\ \mathrm{in}^2$

$$Q := \mathrm{if}\left(\frac{A_e}{A_{g_0}} > 1, 1, \frac{A_e}{A_{g_0}}\right) \quad Q = 1$$

$$F_{cr} := \mathrm{if}\left(\lambda_c \cdot \sqrt{Q} \le 1.5, Q \cdot .658^{Q \cdot \lambda_c^2} F_y, \frac{.877}{\lambda_c^2} F_y\right) \quad F_{cr} = 12.982\ \mathrm{ksi}$$

$f := \phi_c \cdot F_{cr} \quad f = 1.104 \times 10^4 \, psi$

$\phi P_n := if\left(r_{y_0} \geq \frac{KL_3}{200}, f \cdot A_{g_0}, 0\right) \quad \phi P_n = 10.55 \, kip$

$A := if\left(\phi P_n \geq C_3, A_{g_0}, 9999in^2\right) \quad A = 0.956 \, in^2$

$A_{g_3} = min(A)$ for all HSS-shapes

$A_{g_3} = 0.956 \, in^2$

"C" (compression) table:

for each HSS-shape in "C" table:

HSS5.5x5.5x3/16 for example:

$r_{y_0} := 2.16in \quad A_{g_0} := 3.63in^2 \quad t := \frac{3}{16}in \quad d_n := 5.5in$ (nominal depth)

$b := d_n - 3t \quad b = 4.937 \, in$

$\lambda := \frac{b}{t} \quad \lambda = 26.333$

$\phi_c := .85$

for each effective length (similar to AISC Table 4-6):

$KL_4 = 13 \, ft$ for example

$\lambda_c := \frac{KL_4}{r_{y_0} \pi} \sqrt{\frac{F_y}{E}} \quad \lambda_c = 0.916$

$Q := 1$ (assumption)

$F_{cr} := if\left(\lambda_c \cdot \sqrt{Q} \leq 1.5, Q \cdot .658^{Q \cdot \lambda_c^2} F_y, \frac{.877}{\lambda_c^2} F_y\right) \quad F_{cr} = 32.387 \, ksi$

$f := \phi_c \cdot F_{cr} \quad f = 2.753 \times 10^4 \, psi$

$b_e := if\left[\lambda \geq 1.4\sqrt{\frac{E}{f}}, 1.91t\sqrt{\frac{E}{f}}\left(1 - \frac{.381}{\lambda}\sqrt{\frac{E}{f}}\right), b\right] \quad b_e = 4.937 \, in$

$A_e := A_{g_0} - 4\left(b - b_e\right)t \quad A_e = 3.63 \, in^2$

$Q := \text{if}\left(\frac{A_e}{A_{g_0}} > 1, 1, \frac{A_e}{A_{g_0}}\right)$ $Q = 1$

$F_{cr} := \text{if}\left(\lambda_c \cdot \sqrt{Q} \le 1.5, Q \cdot .658^{Q \cdot \lambda_c^2} F_y, \frac{.877}{\lambda_c^2} F_y\right)$ $F_{cr} = 32.387\,\text{ksi}$

$f := \phi_c \cdot F_{cr}$ $f = 2.753 \times 10^4\,\text{psi}$

$\phi P_n := \text{if}\left(r_{y_0} \ge \frac{KL_4}{200}, f \cdot A_{g_0}, 0\right)$ $\phi P_n = 99.931\,\text{kip}$

$A := \text{if}\left(\phi P_n \ge C_4, A_{g_0}, 9999\text{in}^2\right)$ $A = 3.63\,\text{in}^2$

$A_{g_4} = \min(A)$ for all HSS-shapes

$A_{g_4} = 3.63\,\text{in}^2$

"T" (tension) table:

for each HSS-shape in "T" table:

HSS3x3x1/8 for example:

$r_{y_0} := 1.17\text{in}$ $A_{g_0} := 1.3\text{in}^2$

$\phi_t := .9$

$\phi P_n := \phi_t \cdot F_y \cdot A_{g_0}$ $\phi P_n = 5.382 \times 10^4\,\text{lbf}$

$A := \text{if}\left(r_{y_0} \ge \frac{L_5}{300}, \text{if}\left(\phi P_n \ge T_5, A_{g_0}, 9999\text{in}^2\right), 9999\text{in}^2\right)$ $A = 1.3\,\text{in}^2$

$A_{g_5} = \min(A)$ for all HSS-shapes)

$A_{g_5} := \text{if}\left(n_b = 3, A_{g_5}, 0\right)$ $A_{g_5} = 0\,\text{in}^2$

$i := 1$

"C" (compression) table:

for each HSS-shape in "C" table:

HSS4.5x4.5x1/8 for example:

$r_{y_0} := 1.78\text{in}$ $A_{g_0} := 2\text{in}^2$ $t := \frac{1}{8}\text{in}$ $d_n := 4.5\text{in}$ (nominal depth)

$b := d_n - 3t \quad b = 4.125\,\text{in}$

$\lambda := \frac{b}{t} \quad \lambda = 33$

$\phi_c := .85$

for each effective length (similar to AISC Table 4-6):

$KL_{2+4\cdot i} = 9\,\text{ft}$ for example

$\lambda_c := \frac{KL_{2+4\cdot i}}{r_{y_0}\pi}\sqrt{\frac{F_y}{E}} \quad \lambda_c = 0.769$

$Q := 1$ (assumption)

$F_{cr} := \text{if}\left(\lambda_c\cdot\sqrt{Q} \le 1.5, Q\cdot .658^{Q\cdot\lambda_c^2}F_y, \frac{.877}{\lambda_c^2}F_y\right) \quad F_{cr} = 35.91\,\text{ksi}$

$f := \phi_c\cdot F_{cr} \quad f = 3.052 \times 10^4\,\text{psi}$

$b_e := \text{if}\left[\lambda \ge 1.4\sqrt{\frac{E}{f}}, 1.91t\sqrt{\frac{E}{f}}\left(1 - \frac{.381}{\lambda}\sqrt{\frac{E}{f}}\right), b\right] \quad b_e = 4.125\,\text{in}$

$A_e := A_{g_0} - 4\left(b - b_e\right)t \quad A_e = 2\,\text{in}^2$

$Q := \text{if}\left(\frac{A_e}{A_{g_0}} > 1, 1, \frac{A_e}{A_{g_0}}\right) \quad Q = 1$

$F_{cr} := \text{if}\left(\lambda_c\cdot\sqrt{Q} \le 1.5, Q\cdot .658^{Q\cdot\lambda_c^2}F_y, \frac{.877}{\lambda_c^2}F_y\right) \quad F_{cr} = 35.91\,\text{ksi}$

$f := \phi_c\cdot F_{cr} \quad f = 3.052 \times 10^4\,\text{psi}$

$\phi P_n := \text{if}\left(r_{y_0} \ge \frac{KL_{2+4\cdot i}}{200}, f\cdot A_{g_0}, 0\right) \quad \phi P_n = 61.046\,\text{kip}$

$A := \text{if}\left(\phi P_n \ge C_{2+4\cdot i}, A_{g_0}, 9999\text{in}^2\right) \quad A = 2\,\text{in}^2$

$A_{g_{2+4\cdot i}} = \min(A)$ for all HSS-shapes

$A_{g_{2+4\cdot i}} := \text{if}\left[5 + 2(i - 1) = n_b, A_{g_{2+4\cdot i}}, 0\right] \quad A_{g_{2+4\cdot i}} = 0\,\text{in}^2$

"T" (tension) table:

for each HSS-shape in "T" table:

HSS2x2x3/16 for example:

$r_{y_0} := .733\text{in} \quad A_{g_0} := 1.19\text{in}^2$

$\phi_t := .9$

$\phi P_n := \phi_t \cdot F_y \cdot A_{g_0} \qquad \phi P_n = 4.927 \times 10^4\,\text{lbf}$

$$A := \text{if}\left(r_{y_0} \geq \frac{L_{3+4\cdot i}}{300}, \text{if}\left(\phi P_n \geq T_{3+4\cdot i}, A_{g_0}, 9999\text{in}^2\right), 9999\text{in}^2\right) \qquad A = 1.19\,\text{in}^2$$

$A_{g_{3+4\cdot i}} \mathbf{=} \min(A)$ for all HSS-shapes

$$A_{g_{3+4\cdot i}} := \text{if}\left[n_b \leq 3 + 2(i-1), 0, A_{g_{3+4\cdot i}}\right] \qquad A_{g_{3+4\cdot i}} = 1.19\,\text{in}^2$$

"C" (compression) table:

for each HSS-shape in "C" table:

HSS5.5x5.5x1/8 for example:

$r_{y_0} := 2.19\text{in} \quad A_{g_0} := 2.46\text{in}^2 \quad t := \frac{1}{8}\text{in} \quad d_n := 5.5\text{in}$ (nominal depth)

$b := d_n - 3t \quad b = 5.125\,\text{in}$

$\lambda := \frac{b}{t} \quad \lambda = 41$

$\phi_c := .85$

for each effective length (similar to AISC Table 4-6):

$KL_{4+4\cdot i} = 13\,\text{ft}$ for example

$$\lambda_c := \frac{KL_{4+4\cdot i}}{r_{y_0}\pi}\sqrt{\frac{F_y}{E}} \qquad \lambda_c = 0.903$$

$Q := 1$ (assumption)

$$F_{cr} := \text{if}\left(\lambda_c \cdot \sqrt{Q} \leq 1.5, Q \cdot .658^{Q\cdot\lambda_c^{\,2}} F_y, \frac{.877}{\lambda_c^{\,2}} F_y\right) \qquad F_{cr} = 32.698\,\text{ksi}$$

$f := \phi_c \cdot F_{cr} \quad f = 2.779 \times 10^4\,\text{psi}$

$$b_e := \text{if}\left[\lambda \geq 1.4\sqrt{\frac{E}{f}}, 1.91t\sqrt{\frac{E}{f}}\left(1 - \frac{.381}{\lambda}\sqrt{\frac{E}{f}}\right), b\right] \qquad b_e = 5.125\,\text{in}$$

$A_e := A_{g_0} - 4\left(b - b_e\right)t \quad A_e = 2.46\,\text{in}^2$

$Q := \text{if}\left(\frac{A_e}{A_{g_0}} > 1, 1, \frac{A_e}{A_{g_0}}\right) \quad Q = 1$

$F_{cr} := \text{if}\left(\lambda_c\cdot\sqrt{Q} \le 1.5, Q\cdot.658^{Q\cdot\lambda_c{}^2}F_y, \frac{.877}{\lambda_c{}^2}F_y\right) \quad F_{cr} = 32.698\,\text{ksi}$

$f := \phi_c\cdot F_{cr} \quad f = 2.779 \times 10^4\,\text{psi}$

$\phi P_n := \text{if}\left(r_{y_0} \ge \frac{KL_{4+4\cdot i}}{200}, f\cdot A_{g_0}, 0\right) \quad \phi P_n = 68.372\,\text{kip}$

$A := \text{if}\left(\phi P_n \ge C_{4+4\cdot i}, A_{g_0}, 9999\text{in}^2\right) \quad A = 2.46\,\text{in}^2$

$A_{g_{4+4\cdot i}} = \min(A)$ for all HSS-shapes

$A_{g_{4+4\cdot i}} := \text{if}\left[n_b \le 3 + 2(i-1), 0, A_{g_{4+4\cdot i}}\right] \quad A_{g_{4+4\cdot i}} = 2.46\,\text{in}^2$

"T" (tension) table:

for each HSS-shape in "T" table:

HSS3x2x1/4 for example:

$r_{y_0} := .935\text{in} \quad A_{g_0} := 2.21\text{in}^2$

$\phi_t := .9$

$\phi P_n := \phi_t\cdot F_y\cdot A_{g_0} \quad \phi P_n = 9.149 \times 10^4\,\text{lbf}$

$A := \text{if}\left(r_{y_0} \ge \frac{L_{5+4\cdot i}}{300}, \text{if}\left(\phi P_n \ge T_{5+4\cdot i}, A_{g_0}, 9999\text{in}^2\right), 9999\text{in}^2\right) \quad A = 2.21\,\text{in}^2$

$A_{g_{5+4\cdot i}} = \min(A)$ for all HSS-shapes

$A_{g_{5+4\cdot i}} := \text{if}\left[5 + 2(i-1) = n_b, A_{g_{5+4\cdot i}}, 0\right] \quad A_{g_{5+4\cdot i}} = 0\,\text{in}^2$

$i := i + 1 \quad i = 2$

"C" (compression) table:

for each HSS-shape in "C" table:

HSS6x4x3/16 for example:

$r_{y_0} := 1.63\text{in} \quad A_{g_0} := 3.28\text{in}^2 \quad t := \frac{3}{16}\text{in} \quad d_n := 6\text{in}$ (nominal depth)

$b := d_n - 3t \quad b = 5.437\,\text{in}$

$\lambda := \frac{b}{t} \quad \lambda = 29$

$\phi_c := .85$

for each effective length (similar to AISC Table 4-6):

$KL_{2+4\cdot i} = 9\,\text{ft}$ for example

$$\lambda_c := \frac{KL_{2+4\cdot i}}{r_{y_0}\pi}\sqrt{\frac{F_y}{E}} \quad \lambda_c = 0.84$$

$Q := 1$ (assumption)

$$F_{cr} := \text{if}\left(\lambda_c\cdot\sqrt{Q} \le 1.5, Q\cdot .658^{Q\cdot\lambda_c^{2}} F_y, \frac{.877}{\lambda_c^{2}}F_y\right) \quad F_{cr} = 34.238\,\text{ksi}$$

$f := \phi_c\cdot F_{cr} \quad f = 2.91\times 10^4\,\text{psi}$

$$b_e := \text{if}\left[\lambda \ge 1.4\sqrt{\frac{E}{f}}, 1.91t\sqrt{\frac{E}{f}}\left(1 - \frac{.381}{\lambda}\sqrt{\frac{E}{f}}\right), b\right] \quad b_e = 5.437\,\text{in}$$

$A_e := A_{g_0} - 4\left(b - b_e\right)t \quad A_e = 3.28\,\text{in}^2$

$$Q := \text{if}\left(\frac{A_e}{A_{g_0}} > 1, 1, \frac{A_e}{A_{g_0}}\right) \quad Q = 1$$

$$F_{cr} := \text{if}\left(\lambda_c\cdot\sqrt{Q} \le 1.5, Q\cdot .658^{Q\cdot\lambda_c^{2}} F_y, \frac{.877}{\lambda_c^{2}}F_y\right) \quad F_{cr} = 34.238\,\text{ksi}$$

$f := \phi_c\cdot F_{cr} \quad f = 2.91\times 10^4\,\text{psi}$

$$\phi P_n := \text{if}\left(r_{y_0} \ge \frac{KL_{2+4\cdot i}}{200}, f\cdot A_{g_0}, 0\right) \quad \phi P_n = 95.455\,\text{kip}$$

$$A := \text{if}\left(\phi P_n \ge C_{2+4\cdot i}, A_{g_0}, 9999\text{in}^2\right) \quad A = 3.28\,\text{in}^2$$

$A_{g_{2+4\cdot i}} = \min(A)$ for all HSS-shapes)

$A_{g_{2+4\cdot i}} := \text{if}\left[5 + 2(i-1) = n_b, A_{g_{2+4\cdot i}}, 0\right] \quad A_{g_{2+4\cdot i}} = 0\,\text{in}^2$

"T" (tension) table:

for each HSS-shape in "T" table:

HSS2x2x1/8 for example:

$r_{y_0} := .761\text{in} \quad A_{g_0} := .84\text{in}^2$

$\phi_t := .9$

$\phi P_n := \phi_t \cdot F_y \cdot A_{g_0} \qquad \phi P_n = 3.478 \times 10^4\,\text{lbf}$

$$A := \text{if}\left(r_{y_0} \geq \frac{L_{3+4\cdot i}}{300}, \text{if}\left(\phi P_n \geq T_{3+4\cdot i}, A_{g_0}, 9999\text{in}^2\right), 9999\text{in}^2\right) \qquad A = 0.84\,\text{in}^2$$

$A_{g_{3+4\cdot i}} = \min(A)$ for all HSS-shapes

$A_{g_{3+4\cdot i}} := \text{if}\left[n_b \leq 3 + 2(i-1), 0, A_{g_{3+4\cdot i}}\right] \quad A_{g_{3+4\cdot i}} = 0.84\,\text{in}^2$

"C" (compression) table:

for each HSS-shape in "C" table:

HSS4.5x4.5x1/8 for example:

$r_{y_0} := 1.78\text{in} \quad A_{g_0} := 2\text{in}^2 \quad t := \frac{1}{8}\text{in} \quad d_n := 4.5\text{in}$ (nominal depth)

$b := d_n - 3t \quad b = 4.125\,\text{in}$

$\lambda := \frac{b}{t} \quad \lambda = 33$

$\phi_c := .85$

for each effective length (similar to AISC Table 4-6):

$KL_{4+4\cdot i} = 13\,\text{ft}$ for example

$$\lambda_c := \frac{KL_{4+4\cdot i}}{r_{y_0}\pi}\sqrt{\frac{F_y}{E}} \qquad \lambda_c = 1.111$$

$Q := 1$ (assumption)

$$F_{cr} := \text{if}\left(\lambda_c \cdot \sqrt{Q} \leq 1.5, Q \cdot .658^{Q\cdot\lambda_c^2} F_y, \frac{.877}{\lambda_c^2} F_y\right) \qquad F_{cr} = 27.439\,\text{ksi}$$

$f := \phi_c \cdot F_{cr}$ $\quad f = 2.332 \times 10^4\,\text{psi}$

$b_e := \text{if}\left[\lambda \geq 1.4\sqrt{\frac{E}{f}}, 1.91t\sqrt{\frac{E}{f}}\left(1 - \frac{.381}{\lambda}\sqrt{\frac{E}{f}}\right), b\right]$ $\quad b_e = 4.125\,\text{in}$

$A_e := A_{g_0} - 4\left(b - b_e\right)t$ $\quad A_e = 2\,\text{in}^2$

$Q := \text{if}\left(\frac{A_e}{A_{g_0}} > 1, 1, \frac{A_e}{A_{g_0}}\right)$ $\quad Q = 1$

$F_{cr} := \text{if}\left(\lambda_c \cdot \sqrt{Q} \leq 1.5, Q \cdot .658^{Q \cdot \lambda_c{}^2} F_y, \frac{.877}{\lambda_c{}^2} F_y\right)$ $\quad F_{cr} = 27.439\,\text{ksi}$

$f := \phi_c \cdot F_{cr}$ $\quad f = 2.332 \times 10^4\,\text{psi}$

$\phi P_n := \text{if}\left(r_{y_0} \geq \frac{KL_{4+4\cdot i}}{200}, f \cdot A_{g_0}, 0\right)$ $\quad \phi P_n = 46.646\,\text{kip}$

$A := \text{if}\left(\phi P_n \geq C_{4+4\cdot i}, A_{g_0}, 9999\text{in}^2\right)$ $\quad A = 2\,\text{in}^2$

$A_{g_{4+4\cdot i}} = \min(A)$ for all HSS-shapes

$A_{g_{4+4\cdot i}} := \text{if}\left[n_b \leq 3 + 2(i-1), 0, A_{g_{4+4\cdot i}}\right]$ $\quad A_{g_{4+4\cdot i}} = 2\,\text{in}^2$

"T" (tension) table:

for each HSS-shape in "T" table:

HSS6x6x1/8 for example:

$r_{y_0} := 2.39\text{in}$ $\quad A_{g_0} := 2.7\text{in}^2$

$\phi_t := .9$

$\phi P_n := \phi_t \cdot F_y \cdot A_{g_0}$ $\quad \phi P_n = 1.118 \times 10^5\,\text{lbf}$

$A := \text{if}\left(r_{y_0} \geq \frac{L_{5+4\cdot i}}{300}, \text{if}\left(\phi P_n \geq T_{5+4\cdot i}, A_{g_0}, 9999\text{in}^2\right), 9999\text{in}^2\right)$ $\quad A = 2.7\,\text{in}^2$

$A_{g_{5+4\cdot i}} = \min(A)$ for all HSS-shapes

$A_{g_{5+4\cdot i}} := \text{if}\left[5 + 2(i-1) = n_b, A_{g_{5+4\cdot i}}, 0\right]$ $\quad A_{g_{5+4\cdot i}} = 0\,\text{in}^2$

$i := i + 1 \quad i = 3$

"C" (compression) table:

for each HSS-shape in "C" table:

HSS5.5x5.5x3/16 for example:

$r_{y_0} := 2.16\text{in} \quad A_{g_0} := 3.63\text{in}^2 \quad t := \frac{3}{16}\text{in} \quad d_n := 5.5\text{in}$ (nominal depth)

$b := d_n - 3t \quad b = 4.937\ \text{in}$

$\lambda := \frac{b}{t} \quad \lambda = 26.333$

$\phi_c := .85$

for each effective length (similar to AISC Table 4-6):

$KL_{2+4\cdot i} = 9\ \text{ft}$ for example

$\lambda_c := \frac{KL_{2+4\cdot i}}{r_{y_0}\pi}\sqrt{\frac{F_y}{E}} \quad \lambda_c = 0.634$

$Q := 1$ (assumption)

$$F_{cr} := \text{if}\left(\lambda_c\cdot\sqrt{Q} \le 1.5, Q\cdot .658^{Q\cdot\lambda_c^{\ 2}}F_y, \frac{.877}{\lambda_c^{\ 2}}F_y\right) \qquad F_{cr} = 38.88\ \text{ksi}$$

$f := \phi_c\cdot F_{cr} \quad f = 3.305 \times 10^4\ \text{psi}$

$$b_e := \text{if}\left[\lambda \ge 1.4\sqrt{\frac{E}{f}}, 1.91t\sqrt{\frac{E}{f}}\left(1 - \frac{.381}{\lambda}\sqrt{\frac{E}{f}}\right), b\right] \qquad b_e = 4.937\ \text{in}$$

$A_e := A_{g_0} - 4\left(b - b_e\right)t \quad A_e = 3.63\ \text{in}^2$

$$Q := \text{if}\left(\frac{A_e}{A_{g_0}} > 1, 1, \frac{A_e}{A_{g_0}}\right) \qquad Q = 1$$

$$F_{cr} := \text{if}\left(\lambda_c\cdot\sqrt{Q} \le 1.5, Q\cdot .658^{Q\cdot\lambda_c^{\ 2}}F_y, \frac{.877}{\lambda_c^{\ 2}}F_y\right) \qquad F_{cr} = 38.88\ \text{ksi}$$

$f := \phi_c\cdot F_{cr} \quad f = 3.305 \times 10^4\ \text{psi}$

$$\phi P_n := \text{if}\left(r_{y_0} \ge \frac{KL_{2+4\cdot i}}{200}, f\cdot A_{g_0}, 0\right) \qquad \phi P_n = 119.963\ \text{kip}$$

$A := \text{if}\left(\phi P_n \geq C_{2+4\cdot i}, A_{g_0}, 9999\text{in}^2\right) \quad A = 3.63\,\text{in}^2$

$A_{g_{2+4\cdot i}} = \min(A)$ for all HSS-shapes)

$A_{g_{2+4\cdot i}} := \text{if}\left[5 + 2(i-1) = n_b, A_{g_{2+4\cdot i}}, 0\right] \quad A_{g_{2+4\cdot i}} = 3.63\,\text{in}^2$

"T" (tension) table:

for each HSS-shape in "T" table:

HSS1.5x1.5x1/8 for example:

$r_{y_0} := .557\text{in} \quad A_{g_0} := .608\text{in}^2$

$\phi_t := .9$

$\phi P_n := \phi_t \cdot F_y \cdot A_{g_0} \quad \phi P_n = 2.517 \times 10^4\,\text{lbf}$

$A := \text{if}\left(r_{y_0} \geq \frac{L_{3+4\cdot i}}{300}, \text{if}\left(\phi P_n \geq T_{3+4\cdot i}, A_{g_0}, 9999\text{in}^2\right), 9999\text{in}^2\right) \quad A = 0.608\,\text{in}^2$

$A_{g_{3+4\cdot i}} = \min(A)$ for all HSS-shapes)

$A_{g_{3+4\cdot i}} := \text{if}\left[n_b \leq 3 + 2(i-1), 0, A_{g_{3+4\cdot i}}\right] \quad A_{g_{3+4\cdot i}} = 0.608\,\text{in}^2$

"C" (compression) table:

for each HSS-shape in "C" table:

HSS3x3x1/8 for example:

$r_{y_0} := 1.17\text{in} \quad A_{g_0} := 1.3\text{in}^2 \quad t := \frac{1}{8}\text{in} \quad d_n := 3\text{in}$ (nominal depth)

$b := d_n - 3t \quad b = 2.625\,\text{in}$

$\lambda := \frac{b}{t} \quad \lambda = 21$

$\phi_c := .85$

for each effective length (similar to AISC Table 4-6):

$KL_{4+4\cdot i} = 13\,\text{ft}$ for example

$\lambda_c := \frac{KL_{4+4\cdot i}}{r_{y_0}\pi}\sqrt{\frac{F_y}{E}} \quad \lambda_c = 1.69$

$Q := 1$ (assumption)

$$F_{cr} := \text{if}\left(\lambda_c \cdot \sqrt{Q} \le 1.5, Q \cdot .658^{Q \cdot \lambda_c^{\,2}} F_y, \frac{.877}{\lambda_c^{\,2}} F_y\right) \qquad F_{cr} = 14.12\,\text{ksi}$$

$f := \phi_c \cdot F_{cr}$ $f = 1.2 \times 10^4\,\text{psi}$

$$b_e := \text{if}\left[\lambda \ge 1.4\sqrt{\frac{E}{f}}, 1.91t\sqrt{\frac{E}{f}}\left(1 - \frac{.381}{\lambda}\sqrt{\frac{E}{f}}\right), b\right] \qquad b_e = 2.625\,\text{in}$$

$A_e := A_{g_0} - 4\left(b - b_e\right)t$ $A_e = 1.3\,\text{in}^2$

$$Q := \text{if}\left(\frac{A_e}{A_{g_0}} > 1, 1, \frac{A_e}{A_{g_0}}\right) \qquad Q = 1$$

$$F_{cr} := \text{if}\left(\lambda_c \cdot \sqrt{Q} \le 1.5, Q \cdot .658^{Q \cdot \lambda_c^{\,2}} F_y, \frac{.877}{\lambda_c^{\,2}} F_y\right) \qquad F_{cr} = 14.12\,\text{ksi}$$

$f := \phi_c \cdot F_{cr}$ $f = 1.2 \times 10^4\,\text{psi}$

$$\phi P_n := \text{if}\left(r_{y_0} \ge \frac{KL_{4+4\cdot i}}{200}, f \cdot A_{g_0}, 0\right) \qquad \phi P_n = 15.602\,\text{kip}$$

$$A := \text{if}\left(\phi P_n \ge C_{4+4\cdot i}, A_{g_0}, 9999\text{in}^2\right) \qquad A = 1.3\,\text{in}^2$$

$A_{g_{4+4\cdot i}} = \min(A)$ for all HSS-shapes

$$A_{g_{4+4\cdot i}} := \text{if}\left[n_b \le 3 + 2(i-1), 0, A_{g_{4+4\cdot i}}\right] \qquad A_{g_{4+4\cdot i}} = 1.3\,\text{in}^2$$

"T" (tension) table:

for each HSS-shape in "T" table:

HSS23.5x3.5x1/4 for example:

$r_{y_0} := 1.32\text{in}$ $A_{g_0} := 2.91\text{in}^2$

$\phi_t := .9$

$\phi P_n := \phi_t \cdot F_y \cdot A_{g_0}$ $\phi P_n = 1.205 \times 10^5\,\text{lbf}$

$$A := \text{if}\left(r_{y_0} \ge \frac{L_{5+4\cdot i}}{300}, \text{if}\left(\phi P_n \ge T_{5+4\cdot i}, A_{g_0}, 9999\text{in}^2\right), 9999\text{in}^2\right) \qquad A = 2.91\,\text{in}^2$$

$A_{g_{5+4\cdot i}} = \min(A)$ for all HSS-shapes)

$A_{g_{5+4\cdot i}} := \text{if}\left[5+2(i-1) = n_b, A_{g_{5+4\cdot i}}, 0\right] \quad A_{g_{5+4\cdot i}} = 2.91\,\text{in}^2$

weight per foot of each member:

$w_1 := 3.4\,\frac{\text{psi}}{\text{ft}}\,A_{g_1} \quad w_1 = 1.673\,\text{plf}$

$w_2 := 3.4\,\frac{\text{psi}}{\text{ft}}\,\max\left(A_{g_2}, A_{g_6}, A_{g_{10}}, A_{g_{14}}\right) \quad w_2 = 12.342\,\text{plf}$

$w_3 := 3.4\,\frac{\text{psi}}{\text{ft}}\,A_{g_3} \quad w_3 = 3.25\,\text{plf}$

$w_4 := 3.4\,\frac{\text{psi}}{\text{ft}}\,A_{g_4} \quad w_4 = 12.342\,\text{plf}$

$w_5 := 3.4\,\frac{\text{psi}}{\text{ft}}\,\max\left(A_{g_5}, A_{g_9}, A_{g_{13}}, A_{g_{17}}\right) \quad w_5 = 9.894\,\text{plf}$

$i := 1$

$w_{2+4\cdot i} := \text{if}\left[n_b \le 3+2(i-1), 0, 3.4\,\frac{\text{psi}}{\text{ft}}\,\max\left(A_{g_2}, A_{g_6}, A_{g_{10}}, A_{g_{14}}\right)\right] \quad w_{2+4\cdot i} = 12.342\,\text{plf}$

$w_{3+4\cdot i} := 3.4\,\frac{\text{psi}}{\text{ft}}\,A_{g_{3+4\cdot i}} \quad w_{3+4\cdot i} = 4.046\,\text{plf}$

$w_{4+4\cdot i} := 3.4\,\frac{\text{psi}}{\text{ft}}\,A_{g_{4+4\cdot i}} \quad w_{4+4\cdot i} = 8.364\,\text{plf}$

$w_{5+4\cdot i} := \text{if}\left[n_b \le 3+2(i-1), 0, 3.4\,\frac{\text{psi}}{\text{ft}}\,\max\left(A_{g_5}, A_{g_9}, A_{g_{13}}, A_{g_{17}}\right)\right] \quad w_{5+4\cdot i} = 9.894\,\text{plf}$

$i := i + 1 \quad i = 2$

$w_{2+4\cdot i} := \text{if}\left[n_b \le 3+2(i-1), 0, 3.4\,\frac{\text{psi}}{\text{ft}}\,\max\left(A_{g_2}, A_{g_6}, A_{g_{10}}, A_{g_{14}}\right)\right] \quad w_{2+4\cdot i} = 12.342\,\text{plf}$

$w_{3+4\cdot i} := 3.4\,\frac{\text{psi}}{\text{ft}}\,A_{g_{3+4\cdot i}} \quad w_{3+4\cdot i} = 2.856\,\text{plf}$

$w_{4+4\cdot i} := 3.4\,\frac{\text{psi}}{\text{ft}}\,A_{g_{4+4\cdot i}} \quad w_{4+4\cdot i} = 6.8\,\text{plf}$

$w_{5+4\cdot i} := \text{if}\left[n_b \le 3+2(i-1), 0, 3.4\,\frac{\text{psi}}{\text{ft}}\,\max\left(A_{g_5}, A_{g_9}, A_{g_{13}}, A_{g_{17}}\right)\right] \quad w_{5+4\cdot i} = 9.894\,\text{plf}$

$i := i + 1 \quad i = 3$

$w_{2+4\cdot i} := \text{if}\left[n_b \le 3+2(i-1), 0, 3.4\,\frac{\text{psi}}{\text{ft}}\,\max\left(A_{g_2}, A_{g_6}, A_{g_{10}}, A_{g_{14}}\right)\right] \quad w_{2+4\cdot i} = 12.342\,\text{plf}$

$w_{3+4\cdot i} := 3.4\,\frac{psi}{ft}\,A_{g_{3+4\cdot i}} \quad w_{3+4\cdot i} = 2.067\,plf$

$w_{4+4\cdot i} := 3.4\,\frac{psi}{ft}\,A_{g_{4+4\cdot i}} \quad w_{4+4\cdot i} = 4.42\,plf$

$w_{5+4\cdot i} := if\left[n_b \le 3 + 2(i-1), 0, 3.4\,\frac{psi}{ft}\,max\left(A_{g_5}, A_{g_9}, A_{g_{13}}, A_{g_{17}}\right)\right] \quad w_{5+4\cdot i} = 9.894\,plf$

weight of each member:

$P_1 := w_1 h \quad P_1 = 21.746\,lbf$

$P_2 := w_2 s_b \quad P_2 = 123.42\,lbf$

$P_3 := w_3 h \quad P_3 = 42.255\,lbf$

$P_4 := w_4\sqrt{s_b^2 + h^2} \quad P_4 = 202.424\,lbf$

$P_5 := w_5 s_b \quad P_5 = 98.94\,lbf$

$i := 1$

$P_{2+4\cdot i} := w_{2+4\cdot i} s_b \quad P_{2+4\cdot i} = 123.42\,lbf$

$P_{3+4\cdot i} := w_{3+4\cdot i} h \quad P_{3+4\cdot i} = 52.598\,lbf$

$P_{4+4\cdot i} := w_{4+4\cdot i}\sqrt{s_b^2 + h^2} \quad P_{4+4\cdot i} = 137.18\,lbf$

$P_{5+4\cdot i} := w_{5+4\cdot i} s_b \quad P_{5+4\cdot i} = 98.94\,lbf$

$i := i + 1 \quad i = 2$

$P_{2+4\cdot i} := w_{2+4\cdot i} s_b \quad P_{2+4\cdot i} = 123.42\,lbf$

$P_{3+4\cdot i} := w_{3+4\cdot i} h \quad P_{3+4\cdot i} = 37.128\,lbf$

$P_{4+4\cdot i} := w_{4+4\cdot i}\sqrt{s_b^2 + h^2} \quad P_{4+4\cdot i} = 111.528\,lbf$

$P_{5+4\cdot i} := w_{5+4\cdot i} s_b \quad P_{5+4\cdot i} = 98.94\,lbf$

$i := i + 1 \quad i = 3$

$P_{2+4\cdot i} := w_{2+4\cdot i} s_b \quad P_{2+4\cdot i} = 123.42\,lbf$

$P_{3+4\cdot i} := w_{3+4\cdot i} h \quad P_{3+4\cdot i} = 26.874\,lbf$

$P_{4+4\cdot i} := w_{4+4\cdot i}\sqrt{s_b^2 + h^2} \quad P_{4+4\cdot i} = 72.493\,lbf$

$P_{5+4\cdot i} := w_{5+4\cdot i} s_b \quad P_{5+4\cdot i} = 98.94\,\text{lbf}$

deflection - method of virtual work:

virtual unit point load applied to truss:

$P_v := 1\text{kip}$

$$V_v := \frac{P_v}{2} \quad V_v = 0.5\,\text{kip}$$

internal force in each member due to virtual unit point load applied to middle of truss:

$T_{v_1} := 0$

$C_{v_2} := 0$

$C_{v_3} := 0$

$$C_{v_4} := \frac{V_v \cdot \sqrt{{s_b}^2 + h^2}}{h} \quad C_{v_4} = 0.631\,\text{kip}$$

$$T_{v_5} := \frac{C_{v_4} s_b}{\sqrt{{s_b}^2 + h^2}} \quad T_{v_5} = 0.385\,\text{kip}$$

$i := 1$

$$C_{v_{2+4\cdot i}} := \text{if}\left[n_b \le 3 + 2(i-1), 0, T_{v_{1+4\cdot i}}\right] \quad C_{v_{2+4\cdot i}} = 0.385\,\text{kip}$$

$$T_{v_{3+4\cdot i}} := \text{if}\left(n_b = 3, 0, \frac{C_{v_4} h}{\sqrt{{s_b}^2 + h^2}}\right) \quad T_{v_{3+4\cdot i}} = 0.5\,\text{kip}$$

$$C_{v_{4+4\cdot i}} := \text{if}\left(n_b = 3, 0, C_{v_4}\right) \quad C_{v_{4+4\cdot i}} = 0.631\,\text{kip}$$

$$T_{v_{5+4\cdot i}} := \text{if}\left(n_b = 3, 0, C_{v_{2+4\cdot i}} + \frac{C_{v_{4+4\cdot i}} s_b}{\sqrt{{s_b}^2 + h^2}}\right) \quad T_{v_{5+4\cdot i}} = 0.769\,\text{kip}$$

$i := i + 1 \quad i = 2$

$$C_{v_{2+4\cdot i}} := \text{if}\left[n_b \le 3 + 2(i-1), 0, T_{v_{1+4\cdot i}}\right] \quad C_{v_{2+4\cdot i}} = 0.769\,\text{kip}$$

$T_{v_{3+4\cdot i}} := \text{if}\left[n_b \le 3 + 2(i-1), 0, T_{v_{3+4\cdot(i-1)}}\right] \quad T_{v_{3+4\cdot i}} = 0.5\,\text{kip}$

$C_{v_{4+4\cdot i}} := \text{if}\left(n_b = 3, 0, C_{v_4}\right) \quad C_{v_{4+4\cdot i}} = 0.631\,\text{kip}$

$T_{v_{5+4\cdot i}} := \text{if}\left(n_b = 3, 0, C_{v_{2+4\cdot i}} + \frac{C_{v_{4+4\cdot i}} s_b}{\sqrt{s_b^2 + h^2}}\right) \quad T_{v_{5+4\cdot i}} = 1.154\,\text{kip}$

$i := i + 1 \quad i = 3$

$C_{v_{2+4\cdot i}} := \text{if}\left[n_b \le 3 + 2(i-1), 0, T_{v_{1+4\cdot i}}\right] \quad C_{v_{2+4\cdot i}} = 1.154\,\text{kip}$

$T_{v_{3+4\cdot i}} := \text{if}\left[n_b \le 3 + 2(i-1), 0, T_{v_{3+4\cdot(i-1)}}\right] \quad T_{v_{3+4\cdot i}} = 0.5\,\text{kip}$

$C_{v_{4+4\cdot i}} := \text{if}\left(n_b = 3, 0, C_{v_4}\right) \quad C_{v_{4+4\cdot i}} = 0.631\,\text{kip}$

$T_{v_{5+4\cdot i}} := \text{if}\left(n_b = 3, 0, C_{v_{2+4\cdot i}} + \frac{C_{v_{4+4\cdot i}} s_b}{\sqrt{s_b^2 + h^2}}\right) \quad T_{v_{5+4\cdot i}} = 1.538\,\text{kip}$

joint displacement caused by real total load on truss:

$\Delta_1 := \frac{T_{v_1} T_1 h}{A_{g_1} E \cdot P_v} \quad \Delta_1 = 0\,\text{in}$

$\Delta_2 := \frac{C_{v_2} C_2 s_b}{\max\left(A_{g_2}, A_{g_6}, A_{g_{10}}, A_{g_{14}}\right) E \cdot P_v} \quad \Delta_2 = 0\,\text{in}$

$\Delta_3 := \text{if}\left(A_{g_3} = 0, 0, \frac{C_{v_3} C_3 h}{A_{g_3} E \cdot P_v}\right) \quad \Delta_3 = 0\,\text{in}$

$\Delta_4 := \frac{C_{v_4} C_4 \sqrt{s_b^2 + h^2}}{A_{g_4} E \cdot P_v} \quad \Delta_4 = 0.101\,\text{in}$

$\Delta_5 := \frac{T_{v_5} T_5 s_b}{\max\left(A_{g_5}, A_{g_9}, A_{g_{13}}, A_{g_{17}}\right) E \cdot P_v} \quad \Delta_5 = 0.028\,\text{in}$

$i := 1$

$$\Delta_{2+4\cdot i} := \frac{C_{v_{2+4\cdot i}} C_{2+4\cdot i} s_b}{\max\left(A_{g_2}, A_{g_6}, A_{g_{10}}, A_{g_{14}}\right) E\cdot P_v} \qquad \Delta_{2+4\cdot i} = 0.023\,\text{in}$$

$$\Delta_{3+4\cdot i} := \text{if}\left(A_{g_{3+4\cdot i}} = 0, 0, \frac{T_{v_{3+4\cdot i}} T_{3+4\cdot i} h}{A_{g_{3+4\cdot i}} E\cdot P_v}\right) \qquad \Delta_{3+4\cdot i} = 0.109\,\text{in}$$

$$\Delta_{4+4\cdot i} := \text{if}\left(A_{g_{4+4\cdot i}} = 0, 0, \frac{C_{v_{4+4\cdot i}} C_{4+4\cdot i}\sqrt{s_b^2 + h^2}}{A_{g_{4+4\cdot i}} E\cdot P_v}\right) \qquad \Delta_{4+4\cdot i} = 0.106\,\text{in}$$

$$\Delta_{5+4\cdot i} := \frac{T_{v_{5+4\cdot i}} T_{5+4\cdot i} s_b}{\max\left(A_{g_5}, A_{g_9}, A_{g_{13}}, A_{g_{17}}\right) E\cdot P_v} \qquad \Delta_{5+4\cdot i} = 0.098\,\text{in}$$

$i := i + 1$ $i = 2$

$$\Delta_{2+4\cdot i} := \frac{C_{v_{2+4\cdot i}} C_{2+4\cdot i} s_b}{\max\left(A_{g_2}, A_{g_6}, A_{g_{10}}, A_{g_{14}}\right) E\cdot P_v} \qquad \Delta_{2+4\cdot i} = 0.078\,\text{in}$$

$$\Delta_{3+4\cdot i} := \text{if}\left(A_{g_{3+4\cdot i}} = 0, 0, \frac{T_{v_{3+4\cdot i}} T_{3+4\cdot i} h}{A_{g_{3+4\cdot i}} E\cdot P_v}\right) \qquad \Delta_{3+4\cdot i} = 0.093\,\text{in}$$

$$\Delta_{4+4\cdot i} := \text{if}\left(A_{g_{4+4\cdot i}} = 0, 0, \frac{C_{v_{4+4\cdot i}} C_{4+4\cdot i}\sqrt{s_b^2 + h^2}}{A_{g_{4+4\cdot i}} E\cdot P_v}\right) \qquad \Delta_{4+4\cdot i} = 0.078\,\text{in}$$

$$\Delta_{5+4\cdot i} := \frac{T_{v_{5+4\cdot i}} T_{5+4\cdot i} s_b}{\max\left(A_{g_5}, A_{g_9}, A_{g_{13}}, A_{g_{17}}\right) E\cdot P_v} \qquad \Delta_{5+4\cdot i} = 0.183\,\text{in}$$

$i := i + 1$ $i = 3$

$$\Delta_{2+4\cdot i} := \frac{C_{v_{2+4\cdot i}} C_{2+4\cdot i} s_b}{\max\left(A_{g_2}, A_{g_6}, A_{g_{10}}, A_{g_{14}}\right) E\cdot P_v} \qquad \Delta_{2+4\cdot i} = 0.147\,\text{in}$$

$$\Delta_{3+4\cdot i} := \text{if}\left(A_{g_{3+4\cdot i}} = 0, 0, \frac{T_{v_{3+4\cdot i}} T_{3+4\cdot i} h}{A_{g_{3+4\cdot i}} E\cdot P_v}\right) \qquad \Delta_{3+4\cdot i} = 0.043\,\text{in}$$

$$\Delta_{4+4\cdot i} := \text{if}\left(A_{g_{4+4\cdot i}} = 0, 0, \frac{C_{v_{4+4\cdot i}} C_{4+4\cdot i}\sqrt{s_b^2 + h^2}}{A_{g_{4+4\cdot i}} E\cdot P_v}\right) \qquad \Delta_{4+4\cdot i} = 0.04\,\text{in}$$

$$\Delta_{5+4\cdot i} := \frac{T_{v_{5+4\cdot i}} T_{5+4\cdot i} s_b}{\max\left(A_{g_5}, A_{g_9}, A_{g_{13}}, A_{g_{17}}\right) E\cdot P_v} \qquad \Delta_{5+4\cdot i} = 0.26\,\text{in}$$

$$\Sigma\Delta_{TL} := \sum_{i=1}^{5+4\cdot i} \Delta_i \qquad \Sigma\Delta_{TL} = 1.387\,\text{in}$$

trusses - live load applied:

$P_i := w_{LL}\cdot s_t \quad P_i = 1.143 \times 10^4\,\text{lbf}$ (load from interior beams applied to trusses)

$P_e := \frac{w_{LL}\cdot s_t}{2} \quad P_e = 5.714 \times 10^3\,\text{lbf}$ (load from end beams applied to trusses)

$$V := \frac{P_i\cdot\left(n_b - 2\right) + 2P_e}{2} \qquad V = 4.571 \times 10^4\,\text{lbf}$$

internal force in each truss member due to live load applied to truss:

$T_1 := 0$

$C_2 := 0$

$C_3 := P_e \quad C_3 = 5.714\,\text{kip}$

$$C_4 := \frac{\left(V - C_3\right)\cdot\sqrt{s_b^2 + h^2}}{h} \qquad C_4 = 50.465\,\text{kip}$$

$$T_5 := \frac{C_4\cdot s_b}{\sqrt{s_b^2 + h^2}} \qquad T_5 = 30.769\,\text{kip}$$

for each additional double-panel set beyond initial 2-panel truss:

$i := 1$

$$C_{2+4\cdot i} := \text{if}\left[n_b \le 3 + 2(i - 1), 0, T_{5+4(i-1)}\right] \qquad C_{2+4\cdot i} = 30.769\,\text{kip}$$

$$T_{3+4\cdot i} := \text{if}\left[n_b \le 3 + 2(i - 1), 0, \frac{C_{4\cdot i}\cdot h}{\sqrt{s_b^2 + h^2}} - P_i\right] \qquad T_{3+4\cdot i} = 28.571\,\text{kip}$$

$$C_{4+4\cdot i} := \mathrm{if}\left[n_b \le 3+2(i-1), 0, \frac{T_{3+4\cdot i}\cdot\sqrt{s_b^{\,2}+h^2}}{h}\right] \quad C_{4+4\cdot i} = 36.047\,\mathrm{kip}$$

$$T_{5+4\cdot i} := \mathrm{if}\left[n_b \le 3+2(i-1), 0, T_{1+4\cdot i} + \frac{C_{4+4\cdot i}\cdot s_b}{\sqrt{s_b^{\,2}+h^2}}\right] \quad T_{5+4\cdot i} = 52.747\,\mathrm{kip}$$

$i := i + 1 \quad i = 2$

$$C_{2+4\cdot i} := \mathrm{if}\left[n_b \le 3+2(i-1), 0, T_{5+4(i-1)}\right] \quad C_{2+4\cdot i} = 52.747\,\mathrm{kip}$$

$$T_{3+4\cdot i} := \mathrm{if}\left[n_b \le 3+2(i-1), 0, \frac{C_{4\cdot i}\cdot h}{\sqrt{s_b^{\,2}+h^2}} - P_i\right] \quad T_{3+4\cdot i} = 17.143\,\mathrm{kip}$$

$$C_{4+4\cdot i} := \mathrm{if}\left[n_b \le 3+2(i-1), 0, \frac{T_{3+4\cdot i}\cdot\sqrt{s_b^{\,2}+h^2}}{h}\right] \quad C_{4+4\cdot i} = 21.628\,\mathrm{kip}$$

$$T_{5+4\cdot i} := \mathrm{if}\left[n_b \le 3+2(i-1), 0, T_{1+4\cdot i} + \frac{C_{4+4\cdot i}\cdot s_b}{\sqrt{s_b^{\,2}+h^2}}\right] \quad T_{5+4\cdot i} = 65.934\,\mathrm{kip}$$

$i := i + 1 \quad i = 3$

$$C_{2+4\cdot i} := \mathrm{if}\left[n_b \le 3+2(i-1), 0, T_{5+4(i-1)}\right] \quad C_{2+4\cdot i} = 65.934\,\mathrm{kip}$$

$$T_{3+4\cdot i} := \mathrm{if}\left[n_b \le 3+2(i-1), 0, \frac{C_{4\cdot i}\cdot h}{\sqrt{s_b^{\,2}+h^2}} - P_i\right] \quad T_{3+4\cdot i} = 5.714\,\mathrm{kip}$$

$$C_{4+4\cdot i} := \mathrm{if}\left[n_b \le 3+2(i-1), 0, \frac{T_{3+4\cdot i}\cdot\sqrt{s_b^{\,2}+h^2}}{h}\right] \quad C_{4+4\cdot i} = 7.209\,\mathrm{kip}$$

$$T_{5+4\cdot i} := \mathrm{if}\left[n_b \le 3+2(i-1), 0, T_{1+4\cdot i} + \frac{C_{4+4\cdot i}\cdot s_b}{\sqrt{s_b^{\,2}+h^2}}\right] \quad T_{5+4\cdot i} = 70.33\,\mathrm{kip}$$

joint displacement caused by real live load on truss:

$$\Delta_1 := \frac{T_{v_1} T_1 h}{A_{g_1} E\cdot P_v} \quad \Delta_1 = 0\,\mathrm{in}$$

$$\Delta_2 := \frac{C_{v_2} C_2 s_b}{\max\left(A_{g_2}, A_{g_6}, A_{g_{10}}, A_{g_{14}}\right) E\cdot P_v} \quad \Delta_2 = 0\,\mathrm{in}$$

$$\Delta_3 := \text{if}\left(A_{g_3} = 0, 0, \frac{C_{v_3} C_3 h}{A_{g_3} E \cdot P_v}\right) \qquad \Delta_3 = 0\,\text{in}$$

$$\Delta_4 := \frac{C_{v_4} C_4 \sqrt{s_b^{\,2} + h^2}}{A_{g_4} E \cdot P_v} \qquad \Delta_4 = 0.06\,\text{in}$$

$$\Delta_5 := \frac{T_{v_5} T_5 s_b}{\max\left(A_{g_5}, A_{g_9}, A_{g_{13}}, A_{g_{17}}\right) E \cdot P_v} \qquad \Delta_5 = 0.017\,\text{in}$$

$i := 1$

$$\Delta_{2+4 \cdot i} := \frac{C_{v_{2+4 \cdot i}} C_{2+4 \cdot i} s_b}{\max\left(A_{g_2}, A_{g_6}, A_{g_{10}}, A_{g_{14}}\right) E \cdot P_v} \qquad \Delta_{2+4 \cdot i} = 0.013\,\text{in}$$

$$\Delta_{3+4 \cdot i} := \text{if}\left(A_{g_{3+4 \cdot i}} = 0, 0, \frac{T_{v_{3+4 \cdot i}} T_{3+4 \cdot i} h}{A_{g_{3+4 \cdot i}} E \cdot P_v}\right) \qquad \Delta_{3+4 \cdot i} = 0.065\,\text{in}$$

$$\Delta_{4+4 \cdot i} := \text{if}\left(A_{g_{4+4 \cdot i}} = 0, 0, \frac{C_{v_{4+4 \cdot i}} C_{4+4 \cdot i} \sqrt{s_b^{\,2} + h^2}}{A_{g_{4+4 \cdot i}} E \cdot P_v}\right) \qquad \Delta_{4+4 \cdot i} = 0.063\,\text{in}$$

$$\Delta_{5+4 \cdot i} := \frac{T_{v_{5+4 \cdot i}} T_{5+4 \cdot i} s_b}{\max\left(A_{g_5}, A_{g_9}, A_{g_{13}}, A_{g_{17}}\right) E \cdot P_v} \qquad \Delta_{5+4 \cdot i} = 0.058\,\text{in}$$

$i := i + 1 \quad i = 2$

$$\Delta_{2+4 \cdot i} := \frac{C_{v_{2+4 \cdot i}} C_{2+4 \cdot i} s_b}{\max\left(A_{g_2}, A_{g_6}, A_{g_{10}}, A_{g_{14}}\right) E \cdot P_v} \qquad \Delta_{2+4 \cdot i} = 0.046\,\text{in}$$

$$\Delta_{3+4 \cdot i} := \text{if}\left(A_{g_{3+4 \cdot i}} = 0, 0, \frac{T_{v_{3+4 \cdot i}} T_{3+4 \cdot i} h}{A_{g_{3+4 \cdot i}} E \cdot P_v}\right) \qquad \Delta_{3+4 \cdot i} = 0.055\,\text{in}$$

$$\Delta_{4+4 \cdot i} := \text{if}\left(A_{g_{4+4 \cdot i}} = 0, 0, \frac{C_{v_{4+4 \cdot i}} C_{4+4 \cdot i} \sqrt{s_b^{\,2} + h^2}}{A_{g_{4+4 \cdot i}} E \cdot P_v}\right) \qquad \Delta_{4+4 \cdot i} = 0.046\,\text{in}$$

$$\Delta_{5+4 \cdot i} := \frac{T_{v_{5+4 \cdot i}} T_{5+4 \cdot i} s_b}{\max\left(A_{g_5}, A_{g_9}, A_{g_{13}}, A_{g_{17}}\right) E \cdot P_v} \qquad \Delta_{5+4 \cdot i} = 0.108\,\text{in}$$

$i := i + 1 \quad i = 3$

$$\Delta_{2+4\cdot i} := \frac{C_{v_{2+4\cdot i}} C_{2+4\cdot i} s_b}{\max\left(A_{g_2}, A_{g_6}, A_{g_{10}}, A_{g_{14}}\right) E\cdot P_v} \qquad \Delta_{2+4\cdot i} = 0.087\,\text{in}$$

$$\Delta_{3+4\cdot i} := \text{if}\left(A_{g_{3+4\cdot i}} = 0, 0, \frac{T_{v_{3+4\cdot i}} T_{3+4\cdot i} h}{A_{g_{3+4\cdot i}} E\cdot P_v}\right) \qquad \Delta_{3+4\cdot i} = 0.025\,\text{in}$$

$$\Delta_{4+4\cdot i} := \text{if}\left(A_{g_{4+4\cdot i}} = 0, 0, \frac{C_{v_{4+4\cdot i}} C_{4+4\cdot i} \sqrt{s_b^{\,2} + h^2}}{A_{g_{4+4\cdot i}} E\cdot P_v}\right) \qquad \Delta_{4+4\cdot i} = 0.024\,\text{in}$$

$$\Delta_{5+4\cdot i} := \frac{T_{v_{5+4\cdot i}} T_{5+4\cdot i} s_b}{\max\left(A_{g_5}, A_{g_9}, A_{g_{13}}, A_{g_{17}}\right) E\cdot P_v} \qquad \Delta_{5+4\cdot i} = 0.154\,\text{in}$$

$$\Sigma\Delta_{LL} := \sum_{i=1}^{5+4\cdot i} \Delta_i \qquad \Sigma\Delta_{LL} = 0.82\,\text{in}$$

truss deflection check:

$$\Delta_{TL} := \frac{x_t}{240} \qquad \Delta_{TL} = 4\,\text{in}$$

$$\Delta_{LL} := \frac{x_t}{360} \qquad \Delta_{LL} = 2.667\,\text{in}$$

$$\max\left(\Sigma\Delta_{TL} - \Delta_{TL}, \text{if}\left(\Sigma\Delta_{LL} - \Delta_{LL} > 0, \Sigma\Delta_{LL} - \Delta_{LL}, 0\right)\right) = 0\,\text{in} \quad \text{(must be 0)}$$

weight of floor framing:

$$W_b := x_b \cdot w_{sw} \cdot n_b \qquad W_b = 8.666\,\text{kip} \quad \text{(weight of beams)}$$

$$W_t := n_t \cdot \left(2 \sum_{i=1}^{5+4\cdot i} P_i - P_1\right) \qquad W_t = 25.325\,\text{kip} \quad \text{(weight of trusses)}$$

$$W := W_b + W_t \qquad W = 33.991\,\text{kip} \quad \text{(total weight of floor framing)}$$

cost of floor framing:

$$C_b = W_b \cdot \text{if}\left(w_{sw} < 10\text{plf}, 2.09\text{lbf}^{-1}, \text{if}\left(w_{sw} < 20\text{plf}, 1.71\text{lbf}^{-1}, \text{if}\left(w_{se} < 30\text{plf}, 1.46\text{lbf}^{-1}\right)\right)\right)$$

$$\text{if}\left(w_{sw}\right) < 50\text{plf}, 1.34\text{lbf}^{-1}, \text{if}\left(w_{sw}\right) < 90\text{plf}, 1.11\text{lbf}^{-1}, \text{if}\left(w_{sw} < 180\text{plf}, .84\text{lbf}^{-1}, .85\text{lbf}^{-1}\right)\Big)\Big)\Big)\Big)\Big)$$

$C_b = 1.482 \times 10^4$ (cost of beams)

$C_t := W_t \cdot 6\text{lbf}^{-1}$ $C_t = 1.519 \times 10^5$ (cost of trusses)

$C := C_b + C_t$ $C = 1.668 \times 10^5$ (total cost of floor framing)

APPENDIX C

EVOLVER OPTIMIZATION SUMMARY WORKSHEETS

Evolver Optimization Summary Worksheet

	Pratt 100'x100' Area Weight Optimized	Pratt 80'x80' Area Weight Optimized	Pratt 40'x40' Area Weight Optimized
Number of Beams	9	7	5
Number of Trusses	12	12	6
Height of trusses	14	13	8
Cell to Optimize	Sheet1!$M$3	Sheet1!$M$3	Sheet1!$M$3
Optimization Goal	Minimum Value	Minimum Value	Minimum Value
RESULTS			
Valid Trials	453	363	342
Total Recalcs	643	457	455
Original Value	64847.41776	36811.57762	9246.877034
+ soft constraint penalties	0	0	0
= result	64847.41776	36811.57762	9246.877034
Best Value Found	61226.17704	30604.45206	5061.84307
+ soft constraint penalties	0	0	0
= result	61226.17704	30604.45206	5061.84307
Occurred on trial #	190	236	184
Time to find this value	0:03:47	0:02:45	0:02:51
Stopped Because	Halted by User	Halted by User	Halted by User
Optimization Started At	4:42:38 PM	6:30:31 PM	6:55:06 PM
Optimization Finished At	4:53:52 PM	6:35:45 PM	7:00:29 PM
Total Optimization Time	0:08:46	0:04:43	0:05:02
Adjustable Cell	Sheet1!$B$2	Sheet1!$B$2	Sheet1!$B$2
ORIGINAL	13	13	7
BEST	9	7	5
Adjustable Cell	Sheet1!$B$3	Sheet1!$B$3	Sheet1!$B$3
ORIGINAL	12	13	12
BEST	12	12	6
Adjustable Cell	Sheet1!$B$4	Sheet1!$B$4	Sheet1!$B$4
ORIGINAL	13	10	11
BEST	14	13	8
CONSTRAINTS			
Description	Odd Number	Odd Number	Odd Number
Definition	=(Sheet1!$B$2 =ODD(Sheet1!$B$2))	=(Sheet1!$B$2 =ODD(Sheet1!$B$2))	=(Sheet1!$B$2 =ODD(Sheet1!$B$2))
Constraint Type	HARD	HARD	HARD
Satisifed for % of Trials	84.60%	89.28%	87.03%
Satisfied for % of Valid	100.00%	100.00%	100.00%
Penalty Function	N/A	N/A	N/A
Penalty of Best Result	N/A	N/A	N/A
Description	Angle Limitation	Angle Limitation	Angle Limitation
Definition	=(Sheet1!$H$1=0)	=(Sheet1!$H$1=0)	=(Sheet1!$H$1=0)
Constraint Type	HARD	HARD	HARD
Satisifed for % of Trials	78.23%	85.12%	83.96%

Satisfied for % of Valid	100.00%	100.00%	100.00%
Penalty Function	N/A	N/A	N/A
Penalty of Best Result	N/A	N/A	N/A
Description	Deflection Limitation	Deflection Limitation	Deflection Limitation
Definition	=(Sheet1!$H$5=0)	=(Sheet1!$H$5=0)	=(Sheet1!$H$5=0)
Constraint Type	HARD	HARD	HARD
Satisifed for % of Trials	81.18%	88.62%	87.03%
Satisfied for % of Valid	100.00%	100.00%	100.00%
Penalty Function	N/A	N/A	N/A
Penalty of Best Result	N/A	N/A	N/A

ADJUSTABLE CELLS

Description	N/A	N/A	N/A
Solving Method	RECIPE	RECIPE	RECIPE
Number of Time Blocks	N/A	N/A	N/A
Const/Prec Range	N/A	N/A	N/A
Mutation Rate	0.4	0.4	0.4
Crossover Rate	0.5	0.5	0.5
Input Cell/Range Const.	3<=Sheet1!$B$2<=19 [INT]	3<=Sheet1!$B$2<=19 [INT]	3<=Sheet1!$B$2<=19 [INT]
Input Cell/Range Const.	3<=Sheet1!$B$3<=19 [INT]	3<=Sheet1!$B$3<=19 [INT]	3<=Sheet1!$B$3<=19 [INT]
Input Cell/Range Const.	3<=Sheet1!$B$4<=15 [INT]	3<=Sheet1!$B$4<=15 [INT]	3<=Sheet1!$B$4<=15 [INT]

OPTIONS

Population Size	6	6	6
Pause On Error	FALSE	FALSE	FALSE
Graph Progress	TRUE	TRUE	TRUE
Update Display	Never	Never	Never
Log Simulation Data	FALSE	FALSE	FALSE
Random Seed	23917750 (Randomly Chosen)	30388307 (Randomly Chosen)	31867223 (Randomly Chosen)
Stop On Trials	N/A	N/A	N/A
Stop On Minutes	N/A	N/A	N/A
Stop On Change	N/A	N/A	N/A
Stop On Formula	N/A	N/A	N/A

MACROS

Start Of Optimization	N/A	N/A	N/A
Before Calculation	N/A	N/A	N/A
After Calculation	N/A	N/A	N/A
After Storage	N/A	N/A	N/A
End Of Optimization	N/A	N/A	N/A

Evolver Optimization Summary Worksheet

	Pratt 100'x100' Area Cost Optimized	Pratt 80'x80' Area Cost Optimized	Pratt 40'x40' Area Cost Optimized
Number of Beams	9	7	5
Number of Trusses	12	11	5
Height of trusses	14	15	7
Cell to Optimize	Sheet1!$M$5	Sheet1!$M$5	Sheet1!$M$5
Optimization Goal	Minimum Value	Minimum Value	Minimum Value
RESULTS			
Valid Trials	279	2500	838
Total Recalcs	383	3690	1027
Original Value	328499.9583	164940.666	23697.47042
+ soft constraint penalties	0	0	0
= result	328499.9583	164940.666	23697.47042
Best Value Found	328499.9583	162848.3196	22628.61847
+ soft constraint penalties	0	0	0
= result	328499.9583	162848.3196	22628.61847
Occurred on trial #	1	768	114
Time to find this value	0:00:01	0:15:22	0:02:01
Stopped Because	Halted by User	Halted by User	Halted by User
Optimization Started At	10:29:06 PM	8:47:22 PM	8:32:19 PM
Optimization Finished At	10:34:15 PM	9:37:26 PM	8:40:44 PM
Total Optimization Time	0:04:58	0:50:02	0:08:21
Adjustable Cell	Sheet1!$B$2	Sheet1!$B$2	Sheet1!$B$2
ORIGINAL	9	7	5
BEST	9	7	5
Adjustable Cell	Sheet1!$B$3	Sheet1!$B$3	Sheet1!$B$3
ORIGINAL	12	12	6
BEST	12	11	5
Adjustable Cell	Sheet1!$B$4	Sheet1!$B$4	Sheet1!$B$4
ORIGINAL	14	13	8
BEST	14	15	7
CONSTRAINTS			
Description	Odd Number	Odd Number	Odd Number
Definition	=(Sheet1!$B$2 =ODD(Sheet1!$B$2))	=(Sheet1!$B$2 =ODD(Sheet1!$B$2))	=(Sheet1!$B$2 =ODD(Sheet1!$B$2))
Constraint Type	HARD	HARD	HARD
Satisifed for % of Trials	87.73%	82.57%	89.58%
Satisfied for % of Valid	100.00%	100.00%	100.00%
Penalty Function	N/A	N/A	N/A
Penalty of Best Result	N/A	N/A	N/A
Description	Angle Limitation	Angle Limitation	Angle Limitation
Definition	=(Sheet1!$H$1=0)	=(Sheet1!$H$1=0)	=(Sheet1!$H$1=0)
Constraint Type	HARD	HARD	HARD
Satisifed for % of Trials	80.16%	78.51%	88.22%

Satisfied for % of Valid	100.00%	100.00%	100.00%
Penalty Function	N/A	N/A	N/A
Penalty of Best Result	N/A	N/A	N/A
Description	Deflection Limitation	Deflection Limitation	Deflection Limitation
Definition	=(Sheet1!$H$5=0)	=(Sheet1!$H$5=0)	=(Sheet1!$H$5=0)
Constraint Type	HARD	HARD	HARD
Satisifed for % of Trials	84.60%	80.81%	89.58%
Satisfied for % of Valid	100.00%	100.00%	100.00%
Penalty Function	N/A	N/A	N/A
Penalty of Best Result	N/A	N/A	N/A

ADJUSTABLE CELLS

Description	N/A	N/A	N/A
Solving Method	RECIPE	RECIPE	RECIPE
Number of Time Blocks	N/A	N/A	N/A
Const/Prec Range	N/A	N/A	N/A
Mutation Rate	0.4	0.4	0.4
Crossover Rate	0.5	0.5	0.5
Input Cell/Range Const.	3<=Sheet1!$B$2<=19 [INT]	3<=Sheet1!$B$2<=19 [INT]	3<=Sheet1!$B$2<=19 [INT]
Input Cell/Range Const.	3<=Sheet1!$B$3<=19 [INT]	3<=Sheet1!$B$3<=19 [INT]	3<=Sheet1!$B$3<=19 [INT]
Input Cell/Range Const.	3<=Sheet1!$B$4<=15 [INT]	3<=Sheet1!$B$4<=15 [INT]	3<=Sheet1!$B$4<=15 [INT]

OPTIONS

Population Size	6	6	6
Pause On Error	FALSE	FALSE	FALSE
Graph Progress	TRUE	TRUE	TRUE
Update Display	Never	Never	Never
Log Simulation Data	FALSE	FALSE	FALSE
Random Seed	44697920 (Randomly Chosen)	38581068 (Randomly Chosen)	37690638 (Randomly Chosen)
Stop On Trials	N/A	N/A	N/A
Stop On Minutes	N/A	N/A	N/A
Stop On Change	N/A	N/A	N/A
Stop On Formula	N/A	N/A	N/A

MACROS

Start Of Optimization	N/A	N/A	N/A
Before Calculation	N/A	N/A	N/A
After Calculation	N/A	N/A	N/A
After Storage	N/A	N/A	N/A
End Of Optimization	N/A	N/A	N/A

Evolver Optimization Summary Worksheet

	Howe 100'x100' Area Weight Optimized	Howe 80'x80' Area Weight Optimized	Howe 40'x40' Area Weight Optimized
Number of Beams	11	7	5
Number of Trusses	11	12	6
Height of trusses	15	13	8
Cell to Optimize	table!$M$3	table!$M$3	table!$M$3
Optimization Goal	Minimum Value	Minimum Value	Minimum Value

RESULTS

Valid Trials	621	104	232
Total Recalcs	717	151	287
Original Value	63185.89836	30617.72479	5256.086188
+ soft constraint penalties	0	0	0
= result	63185.89836	30617.72479	5256.086188
Best Value Found	60668.99394	30617.72479	5256.086188
+ soft constraint penalties	0	0	0
= result	60668.99394	30617.72479	5256.086188
Occurred on trial #	376	1	1
Time to find this value	0:06:36	0:00:01	0:00:02
Stopped Because	Halted by User	Halted by User	Halted by User
Optimization Started At	10:45:26 PM	8:27:12 PM	8:33:26 PM
Optimization Finished At	10:55:27 PM	8:30:25 PM	8:37:11 PM
Total Optimization Time	0:09:36	0:03:10	0:03:42
Adjustable Cell	table!$B$2	table!$B$2	table!$B$2
ORIGINAL	9	7	5
BEST	11	7	5
Adjustable Cell	table!$B$3	table!$B$3	table!$B$3
ORIGINAL	12	12	6
BEST	11	12	6
Adjustable Cell	table!$B$4	table!$B$4	table!$B$4
ORIGINAL	14	13	8
BEST	15	13	8

CONSTRAINTS

Description	Odd Number	Odd Number	Odd Number
Definition	=(Sheet1!$B$2 =ODD(Sheet1!$B$2))	=(Sheet1!$B$2 =ODD(Sheet1!$B$2))	=(Sheet1!$B$2 =ODD(Sheet1!$B$2))
Constraint Type	HARD	HARD	HARD
Satisifed for % of Trials	92.89%	84.77%	88.85%
Satisfied for % of Valid	100.00%	100.00%	100.00%
Penalty Function	N/A	N/A	N/A
Penalty of Best Result	N/A	N/A	N/A
Description	Angle Limitation	Angle Limitation	Angle Limitation
Definition	=(table!$H$1=0)	=(table!$H$1=0)	=(table!$H$1=0)
Constraint Type	HARD	HARD	HARD
Satisifed for % of Trials	90.66%	77.48%	86.76%

Satisfied for % of Valid	100.00%	100.00%	100.00%
Penalty Function	N/A	N/A	N/A
Penalty of Best Result	N/A	N/A	N/A
Description	Deflection Limitation	Deflection Limitation	Deflection Limitation
Definition	=(table!$H$5=0)	=(table!$H$5=0)	=(table!$H$5=0)
Constraint Type	HARD	HARD	HARD
Satisifed for % of Trials	92.33%	84.77%	88.85%
Satisfied for % of Valid	100.00%	100.00%	100.00%
Penalty Function	N/A	N/A	N/A
Penalty of Best Result	N/A	N/A	N/A

ADJUSTABLE CELLS

Description	N/A	N/A	N/A
Solving Method	RECIPE	RECIPE	RECIPE
Number of Time Blocks	N/A	N/A	N/A
Const/Prec Range	N/A	N/A	N/A
Mutation Rate	0.4	0.4	0.4
Crossover Rate	0.5	0.5	0.5
Input Cell/Range Const.	3<=table!$B$2<=19 [INT]	3<=table!$B$2<=19 [INT]	3<=table!$B$2<=19 [INT]
Input Cell/Range Const.	3<=table!$B$3<=19 [INT]	3<=table!$B$3<=19 [INT]	3<=table!$B$3<=19 [INT]
Input Cell/Range Const.	3<=table!$B$4<=15 [INT]	3<=table!$B$4<=15 [INT]	3<=table!$B$4<=15 [INT]

OPTIONS

Population Size	6	6	6
Pause On Error	FALSE	FALSE	FALSE
Graph Progress	TRUE	TRUE	TRUE
Update Display	Never	Never	Never
Log Simulation Data	FALSE	FALSE	FALSE
Random Seed	45665224 (Randomly Chosen)	7483599 (Randomly Chosen)	7877921 (Randomly Chosen)
Stop On Trials	N/A	N/A	N/A
Stop On Minutes	N/A	N/A	N/A
Stop On Change	N/A	N/A	N/A
Stop On Formula	N/A	N/A	N/A

MACROS

Start Of Optimization	N/A	N/A	N/A
Before Calculation	N/A	N/A	N/A
After Calculation	N/A	N/A	N/A
After Storage	N/A	N/A	N/A
End Of Optimization	N/A	N/A	N/A

Evolver Optimization Summary Worksheet

	Howe 100'x100' Area Cost Optimized	Howe 80'x80' Area Cost Optimized	Howe 40'x40' Area Cost Optimized
Number of Beams	11	9	5
Number of Trusses	11	10	5
Height of trusses	15	14	10
Cell to Optimize	table!$M$5	table!$M$5	table!$M$5
Optimization Goal	Minimum Value	Minimum Value	Minimum Value
RESULTS			
Valid Trials	72	70	269
Total Recalcs	100	97	341
Original Value	316521.9476	331118.7933	24862.92913
+ soft constraint penalties	0	0	0
= result	316521.9476	331118.7933	24862.92913
Best Value Found	316521.9476	160487.6596	23585.59569
+ soft constraint penalties	0	0	0
= result	316521.9476	160487.6596	23585.59569
Occurred on trial #	1	57	176
Time to find this value	0:00:01	0:01:45	0:03:40
Stopped Because	Halted by User	Halted by User	Halted by User
Optimization Started At	8:40:40 PM	4:20:02 PM	8:49:55 PM
Optimization Finished At	8:42:43 PM	4:22:57 PM	8:55:17 PM
Total Optimization Time	0:02:00	0:02:08	0:04:57
Adjustable Cell	table!$B$2	table!$B$2	table!$B$2
ORIGINAL	11	17	5
BEST	11	9	5
Adjustable Cell	table!$B$3	table!$B$3	table!$B$3
ORIGINAL	11	4	6
BEST	11	10	5
Adjustable Cell	table!$B$4	table!$B$4	table!$B$4
ORIGINAL	15	5	8
BEST	15	14	10
CONSTRAINTS			
Description	Odd Number	Odd Number	Odd Number
Definition	=(Sheet1!$B$2 =ODD(Sheet1!$B$2))	=(table!$B$2 =ODD(table!$B$2))	=(Sheet1!$B$2 =ODD(Sheet1!$B$2))
Constraint Type	HARD	HARD	HARD
Satisifed for % of Trials	89.00%	83.51%	88.27%
Satisfied for % of Valid	100.00%	100.00%	100.00%
Penalty Function	N/A	N/A	N/A
Penalty of Best Result	N/A	N/A	N/A
Description	Angle Limitation	Angle Limitation	Angle Limitation
Definition	=(table!$H$1=0)	=(table!$H$1=0)	=(table!$H$1=0)
Constraint Type	HARD	HARD	HARD
Satisifed for % of Trials	78.00%	80.41%	87.68%

Satisfied for % of Valid	100.00%	100.00%	100.00%
Penalty Function	N/A	N/A	N/A
Penalty of Best Result	N/A	N/A	N/A
Description	Deflection Limitation	Deflection Limitation	Deflection Limitation
Definition	=(table!$H$5=0)	=(table!$H$5=0)	=(table!$H$5=0)
Constraint Type	HARD	HARD	HARD
Satisifed for % of Trials	86.00%	83.51%	88.27%
Satisfied for % of Valid	100.00%	100.00%	100.00%
Penalty Function	N/A	N/A	N/A
Penalty of Best Result	N/A	N/A	N/A

ADJUSTABLE CELLS

Description	N/A	N/A	N/A
Solving Method	RECIPE	RECIPE	RECIPE
Number of Time Blocks	N/A	N/A	N/A
Const/Prec Range	N/A	N/A	N/A
Mutation Rate	0.4	0.4	0.4
Crossover Rate	0.5	0.5	0.5
Input Cell/Range Constraint	3<=table!$B$2<=19 [INT]	3<=table!$B$2<=19 [INT]	3<=table!$B$2<=19 [INT]
Input Cell/Range Constraint	3<=table!$B$3<=19 [INT]	3<=table!$B$3<=19 [INT]	3<=table!$B$3<=19 [INT]
Input Cell/Range Constraint	3<=table!$B$4<=15 [INT]	3<=table!$B$4<=15 [INT]	3<=table!$B$4<=15 [INT]

OPTIONS

Population Size	6	6	6
Pause On Error	FALSE	FALSE	FALSE
Graph Progress	TRUE	TRUE	TRUE
Update Display	Never	Never	Never
Log Simulation Data	FALSE	FALSE	FALSE
Random Seed	8304356 (Randomly Chosen)	15287052 (Randomly Chosen)	8851493 (Randomly Chosen)
Stop On Trials	N/A	N/A	N/A
Stop On Minutes	N/A	N/A	N/A
Stop On Change	N/A	N/A	N/A
Stop On Formula	N/A	N/A	N/A

MACROS

Start Of Optimization	N/A	N/A	N/A
Before Calculation	N/A	N/A	N/A
After Calculation	N/A	N/A	N/A
After Storage	N/A	N/A	N/A
End Of Optimization	N/A	N/A	N/A

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