Chapter XI. The Role of Electrodes Physical Aspects: Comparison of Rectangular and Cylindrical Geometrical Models

11.1 Introduction and Motivation

To practitioners there is no doubt that electroremediation is a promising technology for heavy metals extraction, as well as for some organic compounds, from a soil matrix. Despite that, there are a number of shortcomings, detected during field testing, that have raised some concerns and issues to be solved. Unable to properly control process efficiency, practitioners can know with certainty neither what variables drive the operation nor how well they may be optimized. To some extent, extraction rate and efficiency have been associated with soil type, grain size, pollutant concentration, physicochemical properties and many other characteristics leading to important research on how to deal with these unknowns. In other words, most of the efforts have been directed to the site and contaminant characteristics as the source of problem rather than studying the effect of the applying technology.

Conclusive studies developed by the US Army Environmental Center (US Army, 2000) textually recognize that the role of some technology aspects is not yet clearly understood. The following statement expresses that:

> There is a lack of understanding of the **impact that electrode shape and electrode placement** will have on the electric field shape and intensity formed within the soil matrix. The electric field shape and intensity will affect the formation of and **movement of mobile heavy metal species**. If complete coverage of treatment area is not achieved, then all of the contaminant may not be extracted.

The research implications of the above statement are many and they can be systematically studied. For example, electrode shapes and placements have direct influence on the hydrodynamics taking place near the electrode zone. Furthermore, if the electrical field is modified, typical mechanisms such as electroosmosis and electromigration may not behave as currently observed and predicted. In addition, electrostatic variation may lead to uncontrollable temperature changes due to Joule heating. All these implications are just a brief analysis of what needs to be addressed in a formal study. Consequently, a systematic approach is needed to determine the role of electrode shapes in the electroremediation process. In fact, the collection of topics addressed in published articles, dealing with electrokinetic cells, allows concluding that no research has been devoted to analyze the role played by the electrode physical aspect.

From several options, mathematical models have been proposed as useful tools for analysis. On the other hand, more research still needs to be done in order to determine the effect of electrode shapes on process efficiency. Therefore, and considering that two mathematical models have been previously developed for the region near the electrode zone (i.e. rectangular and cylindrical geometrical aspects), this present chapter focuses on comparing the main characteristics that may be found in two electrodes of different shapes. The main driving idea is the identification of features that may promote better extraction protocols as well as process efficiencies. This information, still missing in the literature, may have meaningful value among the practitioners.

11.2 Basis of the Comparison and Analysis

The four most developed electroremediation technologies, by design, use specific electrode shapes. Cation Selective Membrane and Lasagna methods are associated with rectangular electrodes while Ceramic Casting and Electrochemical Ion Exchange use cylindrical devices. Since no other geometrical aspects have been reported on electrode shapes, rectangular and cylindrical devices are the main focus of the present comparison (see figure 11.1). The comparison of results given by these individual geometries is needed to further understand the role played by physical aspects. This simple geometrical approach is the basis for the study of the hydrodynamics taking place near the electrode zone under the stress of an electrical field as in an electrokinetic remediation cell. In particular, the hydrodynamics predicted by using the planar and cylindrical electrode geometrical models are compared and analyzed in this section. The main idea in the analysis is to find prediction discrepancies, when using these two geometrical models, that could facilitate determining under what conditions possible flow regimes may favor clean up processes.

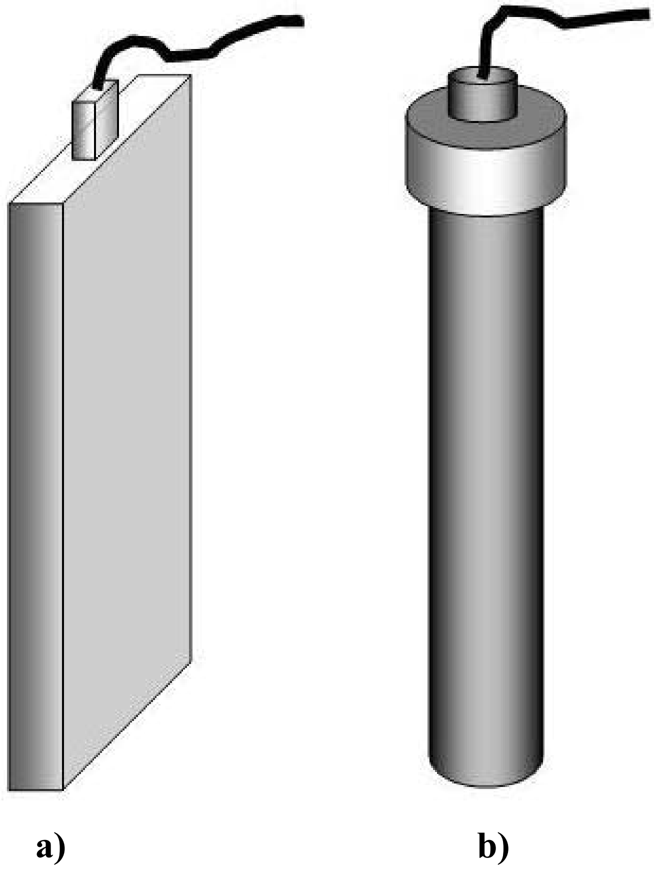

Figure 11.1 Electrodes sketch of rectangular **a)** and cylindrical **b)** geometrical aspect used in the analysis.

For simplicity the hydrodynamics near the electrode zone have been studied using a boundary layer approach. The fundamental analysis of such systems, i.e. rectangular and cylindrical, was developed in the preceding two chapters. Under these conditions, an electrostatic field of constant value has been granted in the radial or transversal direction whose main influence is the Joule heating effect, presented by the fluid within the domain of the boundary layer. This influence requires the analysis of the heat transfer equation, momentum and continuity equation. In consequence, the following subsections deal with the main features of the model equations describing the system under study. In each of these subsections and for the purpose of context, only the most relevant expressions of each model are presented. The detailed description and formulation of these models can be found in those chapters already referenced above. In particular, the discussion will concentrate on the comparison of results among models, as well as on the implications of such results.

11.3 The Heat Transfer Problem

The basis for considering a heat transfer problem near the electrode zone has two basic sources. First, the temperature of the electrode surface may be different than the fluid temperature, away from the boundary layer, promoting temperature changes within the layer domain. The second source is the electric resistance presented by the fluid in the boundary layer governed by the Joule heating effect. The increase of temperature in the system usually affects fluid properties. When density is affected, temperature changes yield buoyancy-driven type of flow modifying the hydrodynamics of the system under the thermal stress. It is for this reason that knowing the different temperature profiles promoted by the two geometrical aspects is of very much interest. The following table (Table 11.1) presents a summary of the fundamental equations for heat transfer and the continuity equation that couple to them to mathematically describe the systems in comparison.

Table 11.1 Comparison of heat transfer and continuity equations describing the electrode systems of different geometrical aspects

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Planar geometrical aspect model

$$v_x^+ \cdot \frac{\partial \theta}{\partial \xi} + v_y^+ \cdot \frac{\partial \theta}{\partial \eta} = \frac{\partial^2 \theta}{\partial \eta^2} + \phi^2 \qquad \text{(Heat)}$$

$$\frac{\partial V_x^+}{\partial \xi} + \frac{\partial V_y^+}{\partial \eta} = 0 \qquad \text{(Continuity)}$$

Cylindrical geometrical aspect model

$$v_x^+ \cdot \frac{\partial \theta}{\partial \xi} + v_r^+ \cdot \frac{\partial \theta}{\partial \eta} = \frac{1}{\eta} \cdot \frac{\partial}{\partial \eta}\left(\eta \cdot \frac{\partial \theta}{\partial \eta}\right) + \phi^2 \qquad \text{(Heat)}$$

$$\frac{\partial V_x^+}{\partial \xi} + \frac{1}{\eta} \cdot \frac{\partial}{\partial \eta}\left(\eta \cdot \frac{\partial V_r^+}{\partial \eta}\right) = 0 \qquad \text{(Continuity)}$$

The heat transfer equations present the same type of expressions in the axial direction; however, the transversal or radial component shows important differences due to geometry. In a term-to-term basis, the cylindrical model has an additional expression; this is $\frac{1}{\eta} \cdot \left(\frac{\partial \theta}{\partial \eta}\right)^2$. This term introduces square variations of temperature in the radial direction by an inverse factor of the same coordinate to the entire heat balance in comparison to the rectangular model. A comparison of the continuity equations brings up the same result. The practical implications of the just described differences have been observed on how Joule heating affects temperature profiles for these two geometrical aspects (please see figure 9.2 and 10.2 in the previous chapters). In rectangular devices, for instance, it is expected that fluid electrical resistance will promote higher temperatures profiles in the axial and transversal direction. This behavior is completely opposite in cylindrical devices. The effect of the geometry term, $\frac{1}{\eta}\cdot\left(\frac{\partial\theta}{\partial\eta}\right)^2$, modify the predictions to a contracted temperature front instead of a further reaching one as in rectangular geometry. In other words, temperature developments are better controlled with cylindrical devices in the region near the electrode. Rectangular devices increase and propagate temperature, under Joule heating, invading the treatment zone. If this is the case, buoyancy driven flows may predominate over electroosmotic and pressure driven flows significantly affecting removal efficiency.

11.4 The Boundary Layer

Temperature developments have two direct consequences accounted in this comparison. These are the effects on velocity profiles and boundary layer. This sub-section concentrates on the analysis of boundary layer while the former will be discussed last. To that end, it can be stated that the boundary layer's thickness validates the non slippery boundary condition if the thickness value is close to zero. This common assumption may be affected by temperature differences between the electrode surface and the fluid temperature, away from the boundary layer, or by fluid electric resistance, Joule heating effect. If that is the case, boundary layer thicknesses may vary critically invading (or evacuating) important treatment sections. This is without mentioning the change of governing conditions that the phenomenon may cause on the affected region.

To introduce the comparison, Table 11.2 presents the summary of the differential equation system and boundary conditions that lead to the numerical computing of the boundary layer thickness in both cases, i.e., rectangular and cylindrical devices. A preliminary inspection of the corresponding expressions for boundary layers clearly indicates the complexity of terms found in the cylindrical model. Comparable terms, between geometries, have opposite signs which already suggests opposite results. This predicted differences can be observed in Figures 9.3 and 10.3 (please see chapters IX and X).

Table 11.2 Comparison of final differential equations for boundary layer, amplitude velocity and boundary condition describing the electrode systems of different geometrical aspects

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Rectangular geometrical aspect model

Boundary Layer

$$\frac{\partial \delta^+}{\partial \xi} = \left(\frac{105}{\mathrm{Re}} + 120\right)\left(\frac{1}{U^+ \cdot \delta^+}\right) + 70 \cdot \frac{\mathrm{Gr}}{\mathrm{Re}^2}\left(\frac{\delta^+}{U^{+2}}\right) - 60 \cdot \phi^2 \cdot \frac{\delta^+}{U^+}$$

Amplitud Velocity

$$\frac{\partial U^+}{\partial \xi} = -\left(\frac{105}{\mathrm{Re}} + 60\right)\left(\frac{1}{\delta^{+2}}\right) - 70 \cdot \frac{\mathrm{Gr}}{\mathrm{Re}^2}\left(\frac{1}{U^+}\right) + 30 \cdot \phi^2$$

Boundary Conditions

$\delta^+ = 0 \quad @ \quad \xi = 0$

$U^+ = 0 \quad @ \quad \xi = 0$

Cylindrical geometrical aspect model

Boundary Layer

$$\frac{\partial \delta^+}{\partial \xi} = \frac{60}{U^+} \cdot (G(U^+, \delta^+, b) + \delta^+ \cdot \phi^2) - \frac{105}{U^{+2}} \cdot F(U^+, \delta^+, b) - \frac{35}{U^{+2}} \cdot \frac{\mathrm{Gr}}{\mathrm{Re}^2} \cdot \delta^+$$

Amplitud Velocity

$$\frac{\partial U^+}{\partial \xi} = -\frac{30}{\delta^+} \cdot (G(U^+, \delta^+, b) + \delta^+ \cdot \phi^2) + \frac{105}{U^+ \cdot \delta^+} \cdot F(U^+, \delta^+, b) + \frac{35}{U^+} \cdot \frac{\mathrm{Gr}}{\mathrm{Re}^2}$$

Boundary Conditions

$\delta^+ = 0 \quad @ \quad \xi = 0$

$U^+ = 0 \quad @ \quad \xi = 0$

The behavior observed in the boundary layer under the Joule heating effect is similar to that observed in temperature profiles. In other words, temperature developments due to higher Joule heating values increase the boundary layer thickness in rectangular devices. This phenomenon develops a growing ascending bulk of fluid pushing in the transversal direction. On the contrary, the boundary layer becomes thinner in the cylindrical devices under the same conditions. Little growth is expected in the axial direction with higher heating effects. The practical use of this information suggests that a higher electrical field can be applied with cylindrical electrodes rather than with rectangular devices for which the boundary layer thickness experiences an uncontrollable swelling effect as the surface is approached (along the axial direction). However, this negative effect, observed in rectangular electrodes, can be advantageously used to boost up electroosmotic or pressure driven flows in the direction of the opposite electrode.

11.5 The Hydrodynamic Problem

The hydrodynamics taking place near the electrode region are described in fundamental terms by the momentum equation. As has been established, buoyancy driven forces induce movement on bulk fluid due to density variations. The specific hydrodynamic behavior needs to be determined as well as its influence on process efficiency for the most typical cases of electrode geometrical aspect. The momentum equations for the two typical cases under study are summarized in Table 11.3.

When comparing the rectangular and cylindrical models, the effect of the geometry term concept, $\frac{1}{\eta}.\left(\frac{\partial\theta}{\partial\eta}\right)^2$, used in the comparison of heat and continuity is an important factor to consider in the analysis of momentum as well. Repeatedly, the axial components of the rectangular and cylindrical models are the same. Conversely, the transversal component in rectangular devices presents a displacement with respect to the radial component in cylindrical devices equivalent to the squared variations of axial velocity in the radial direction by an inverse factor of the same coordinate. This is $\frac{1}{\eta}\cdot\left(\frac{\partial v_X^{+}}{\partial\eta}\right)^2$.

Table 11.3 Comparison of momentum equations describing the electrode systems of different geometrical aspects

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Rectangular geometrical aspect model

$$v_x^{+}\cdot\frac{\partial v_x^{+}}{\partial\xi}+v_y^{+}\cdot\frac{\partial v_x^{+}}{\partial\eta}=\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v_x^{+}}{\partial\eta^2}\right)+\frac{\mathrm{Gr}}{\mathrm{Re}^2}\theta \qquad \text{(Momentum)}$$

Cylindrical geometrical aspect model

$$v_x^{+}\cdot\frac{\partial v_x^{+}}{\partial\xi}+v_r\cdot\frac{\partial v_x^{+}}{\partial\eta}=\frac{1}{\mathrm{Re}}\cdot\frac{1}{\eta}\cdot\frac{\partial}{\partial\eta}\left(\eta\cdot\frac{\partial v_x^{+}}{\partial\eta}\right)+\frac{\mathrm{Gr}}{\mathrm{Re}^2}\theta \qquad \text{(Momentum)}$$

In terms of axial velocity, a simple comparison of figures 9.4a and 10.4a indicates that temperature developments due to joule heating enlarge the velocity profiles reaching higher maximum values and vanishing further in the transversal direction of rectangular devices. In contrast, the same stress causes a contraction of the axial velocity profile in the radial direction (wide) with higher maximum axial values. In practical terms, rectangular devices offer larger areas for fluid collection than cylindrical electrodes. However, fluid delivery may be an issue with rectangular electrodes especially if they are affected by thermal stress. From another view, the Joule heating phenomenon causes a reduction in the effective area of treatment with rectangular devices as the axial velocity profile grows in the transversal direction. This particular effect is in agreement with the predictions obtained in the heat transfer analysis above. In other words, rectangular devices promote buoyancy driven flows over electroosmotic and pressure driven flows which may have a significant effect on removal efficiency. In those cases it is not rare to observe canalization at the surface of the treated area between electrodes.

From the analysis above, there are several characteristics that many favor the use of one particular electrode geometrical aspect over the other. However, there is one additional characteristic that has not yet been discussed and that it may bring important information as to decide what electrode aspect will be more effective. It has been indicated that rectangular electrodes may have some advantages over cylindrical devices; however, those features can be emulated on cylindrical electrodes changing the radial ratio aspect. In other words, shorter electrodes will behave to some extent as rectangular devices but this is not reciprocal. This may suggest that cylindrical devices offer the most practical features.

11.6 Summary of the Chapter

The present chapter focuses on comparison of the most typical electrode shape used in electrokinetic applications, i.e. rectangular and cylindrical. The basis of the comparison is the analysis of the heat transfer, boundary layer and momentum problems. In particular, the predictive results obtained in chapter IX and X for temperature, boundary layer and axial velocity profiles are compared across the geometries studied.

The analysis brings up important geometrical aspects by simple inspection of the fundamental equations describing the electrode region system. For example, two effects of geometry terms has been identified; these are $\frac{1}{\eta}.\left(\frac{\partial\theta}{\partial\eta}\right)^2$ and $\frac{1}{\eta}.\left(\frac{\partial V_X{}^+}{\partial\eta}\right)^2$. Both terms suggest that under similar conditions opposite results will be obtained in the transversal or radial direction. This has been corroborated by a numerical plot of the pertinent equations.

Practical implications of the analysis are discussed to illustrate the better use of rectangular or cylindrical devices. Special emphasis is given to the effect on removal efficiency under Joule heating. The knowledge acquired in this study is useful to promote a deeper understanding of the role of the electrode shape in electroremediation cells design.

Finally, this is the closing chapter of the fourth part, “Hydrodynamics Aspects of the Electrokinetic Remediation Process in the near the Electrodes Zone: Boundary Layer Approach to Electro-Mechanisms and Joule Heating,” of the present thesis.